Understanding the Understanding Moon Phases Video

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RIME

ASU

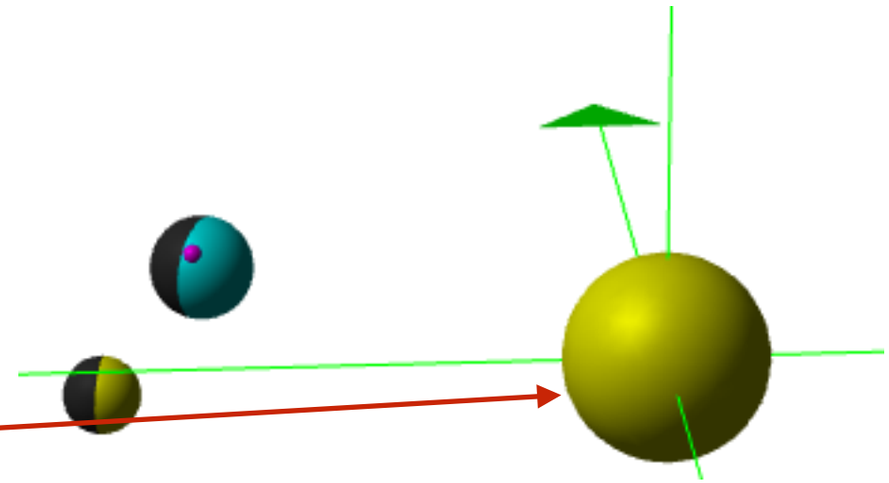
$$\text{MoonOrbitRad} = \text{slider}(0.1, 1) \quad = 0.6625$$

$$\text{MoonOrbitsPerYr} = \text{slider}(1, 12, 11) \quad = 4$$

$$\text{DaysPerYr} = \text{slider}(10, 60, 50) \quad = 20$$

$$\text{Inclination} = \text{slider}(0, 0.75, 75) \quad = 0.5$$

Sliders to set parameters in animation. *Inclination* adjusts the inclination of Moon's orbit to Earth's equator.



$$\left\| \begin{bmatrix} x \\ y \\ z \end{bmatrix} \right\| = 0.4$$

All points in 3D coordinate system at a distance of 0.4 from (0,0,0).

$$MR = 0.15,\ ER = 0.2,\ SR = \frac{ER}{5}$$

Moon radius, Earth radius, and radius of spot on Earth.

$$EC = \begin{bmatrix} 2\cos 2\pi n \\ 2\sin 2\pi n \\ 0 \end{bmatrix}$$

Earth center: n is an automatic slider varying from 0 to 1. Effect is to rotate Earth's center in a circle within the x-y plane at a distance of 2 from (0,0,0).

$$MC = EC + \text{MoonOrbitRad} \begin{bmatrix} \cos 2\,\text{MoonOrbitsPerYr} \cdot \pi n \\ \sin 2\,\text{MoonOrbitsPerYr} \cdot \pi n \\ \text{Inclination} \sin(2\,\text{MoonOrbitsPerYr} \cdot \pi n + 1) \end{bmatrix}$$

Defines Moon's center as an offset from Earth's center

$$LSmoon = MR \begin{bmatrix} (\sin \pi u)\ \cos \pi v \\ (\sin \pi u)\ \sin \pi v \\ \cos \pi u \end{bmatrix} + MC \qquad DSmoon = MR \begin{bmatrix} -(\sin \pi u)\ \cos \pi v \\ -(\sin \pi u)\ \sin \pi v \\ \cos \pi u \end{bmatrix} + MC$$

$$LSearth = ER \begin{bmatrix} (\sin \pi u)\ \cos \pi v \\ (\sin \pi u)\ \sin \pi v \\ \cos \pi u \end{bmatrix} + EC \qquad DSearth = ER \begin{bmatrix} -(\sin \pi u)\ \cos \pi v \\ -(\sin \pi u)\ \sin \pi v \\ \cos \pi u \end{bmatrix} + EC$$

Define half-spheres with respective radii for lit and dark sides of Moon and Earth. Half spheres are centered at respective centers. Boundaries of half spheres are parallel to the y-z plane. Parameters u and v vary from 0 to 1 independently.

$a = \text{slider}(0, 1)$ a = 0.15

An "experimental" slider to adjust Spot's location on Earth's surface

$$\text{Spot} = \text{SR}\begin{bmatrix}(\sin 2\pi u)\cos 2\pi v\\(\sin 2\pi u)\sin 2\pi v\\\cos 2\pi u\end{bmatrix} + 0.9\text{ER}\begin{bmatrix}(\sin 2\pi a)\cos 2\pi 0.5\\(\sin 2\pi a)\sin 2\pi 0.5\\\cos 2\pi a\end{bmatrix} + \text{EC}$$

Defines a sphere of radius *SR* centered at a point just short of Earth's surface.

$R_z(\text{LSmoon}, \text{taninv}(\text{yCor}(\text{MC}), \text{xCor}(\text{MC})) + \frac{\pi}{2}, \text{MC})$

$R_z(\text{DSmoon}, \text{taninv}(\text{yCor}(\text{MC}), \text{xCor}(\text{MC})) + \frac{\pi}{2}, \text{MC})$

Rotate all points on lit or dark side of moon around Moon Center through an angle equal to the direction of of MC from the polar axis in the *x-y* plane. Add π/2 to rotation angle so lit side faces the sun and dark side faces away from the sun.

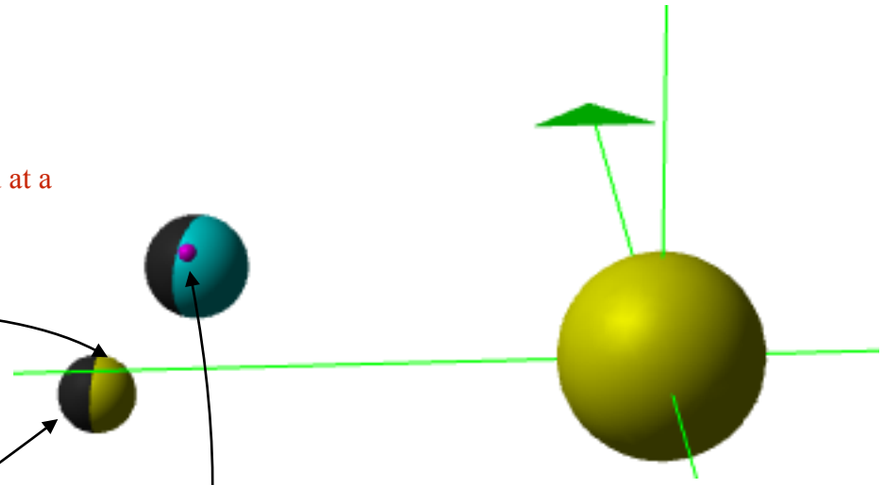
$R_z(\text{LSearth}, \text{taninv}(\text{yCor}(\text{EC}), \text{xCor}(\text{EC})) + \frac{\pi}{2}, \text{EC})$

$R_z(\text{DSearth}, \text{taninv}(\text{yCor}(\text{EC}), \text{xCor}(\text{EC})) + \frac{\pi}{2}, \text{EC})$

Rotate all points on lit or dark side of Earth around Earth Center through an angle equal to the direction of of EC from the polar axis in the *x-y* plane. Add π/2 to rotation angle so lit side faces the sun and dark side faces away from the sun..

$R_z(\text{Spot}, \text{DaysPerYr}\,2\pi n, \text{EC})$

Rotate Spot around Earth Center through an angle adjusted for number of days per year in one earth orbit..

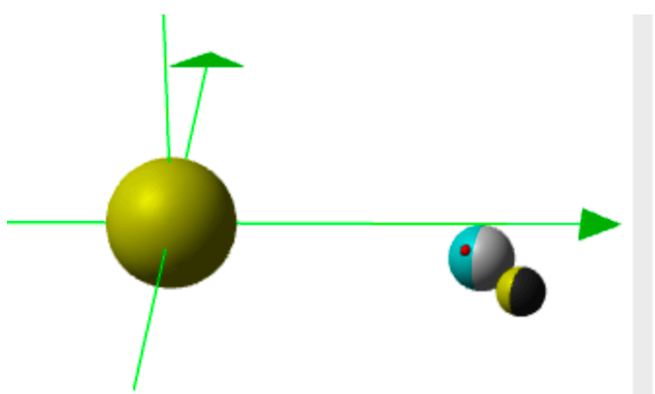


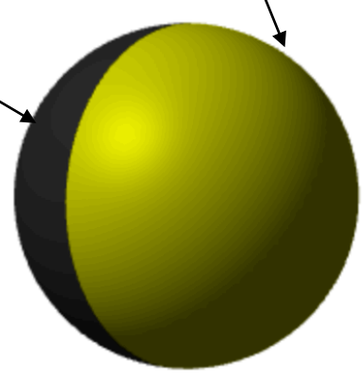
$$\begin{bmatrix}x'\\y'\\z'\end{bmatrix} = 3R_z(\text{LSmoon} - \text{MC}, -(\text{MOY} - 1)\,2\pi n + \pi, O)$$

Graph the lit side of the moon in the right pane, centered at the origin and rotated according to its position relative to Earth.

$$\begin{bmatrix}x'\\y'\\z'\end{bmatrix} = 3R_z(\text{DSmoon} - \text{MC}, -(\text{MOY} - 1)\,2\pi n + \pi, O)$$

Graph the dark side of the moon in the right pane, centered at the origin and rotated according to its position relative to Earth.





$$R_z(X, \theta, \text{Center}) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} (X - \text{Center}) + \text{Center}$$

Function to rotate point X through angle θ around a line through Center parallel to the z-axis..

$$\text{xCor}(X) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \cdot X$$

Dot product to extract x-coordinate of point X.

$$\text{yCor}(X) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \cdot X$$

Dot product to extract y-coordinate of point X.

$$\text{taninv}(y, x) = \begin{cases} \text{atan}\left(\frac{y}{x}\right) \text{ if And}(x > 0, y \geq 0) \\ \text{atan}\left(\frac{y}{x}\right) + \pi \text{ if } x < 0 \\ \text{atan}\left(\frac{y}{x}\right) + 2\pi \text{ if And}(x > 0, y < 0) \\ \frac{\pi}{2} \text{ if And}(x = 0, y > 0) \\ \left(3\frac{\pi}{2} \text{ if And}(x = 0, y < 0)\right) \end{cases}$$

Function to determine direction from (0,0) of point in the x-y plane having coordinates (x,y).