Sound is off at first.

Bill: Could you go through that once more and show me how you figured out his time [*puts 40 in for the Turtle-Over Box*]. I just want to make sure that I understand that you understand. Okay, let's get this one out of here. Put a zero [*types 0 for the Rabbit-speed Box*].

Ann: Okay.

Bill: You can use the paper there [*points to a pile of scratch paper*] and a pencil.

Ann: I would … I would take forty, right?

Bill: Uh huh.

Ann: [*Uses the mouse as an on-screen pointer*] I've got forty there [*Turtle-Over Box*] and I've got thirty here [*Turtle-Back Box*], so I would … I would, um, divide forty into a hundred … which would come up with … eighty … er, it could go in twice.

Bill: Uh huh.

Ann: And then,

Bill: What is that, the twice [*holds up two fingers*]? What is that? What does each of those twices represent, I should say [*holds up two fingers again and shakes them*]?

Ann: Forty. They each represent forty.

Bill: In distance, they represent that. But what you said, "Goes in there two times." What are those two [*taps desk twice*]? They're not feet are they?

Ann: No, they're … forty feet.

Bill: Okay. The forty goes into a hundred … two times [*moves finger over on desk*].

Ann: [*Begins to fidget*] Yeah.

Bill: What does that number two [*holds up two fingers*] … Let's just stop right there and figure it out.

Ann: Eighty feet.

Bill: No.

Ann: Eighty feet.

Bill: No, that's the distance.

Ann: Yeah! [*Smiles*].

Bill: But the two [*holds out two fingers again*] represents something else.

Ann: The two … I don't know.

Bill: What happened when you went that first forty feet.

Ann: … It was one second.

Bill: Okay, what happened when you went the second forty feet?

Ann: Two seconds.

Bill: Okay. When you divided the forty into the hundred and you say it goes in there two times plus some, what else do those two represent [*holds up two fingers*]?

Ann: Seconds?

Bill: Don't they?

Ann: [*Pause*] Yeah.

Bill: Let's go through the example. Do you need a pencil or a pen? I'll give you a pen. I don't have a pencil, but you can use the pen there. Here's some paper. Tell me what your calculations are on that one and we'll discuss it some.

Ann: Okay, forty went into a hundred, right? It goes in two seconds. That would be eighty. [*Pause*] And that leaves twenty left over.

Bill: Uh huh.

Ann: So nothing else would come down so you put a point there,

Bill: There you go.

Ann: and zero here.

Bill: Uh huh.

Ann: And you put [*inaudible*] here.

Bill: Good!

Ann: And bring this down. So you'd have 200.

Bill: Uh huh.

Ann: Forty goes into 200 how many times? That's what--if it goes in 80 here, it has to go in 2 more times down here [*surmising that 200÷40 is approximately 4*].

Bill: Okay.

Ann: That would be …

Bill: Well let's just say how many times [*covers up the last digits of 40 and 200*] will 4 go into 20?

Ann: Four goes into twenty five times.

Bill: Okay. Let's stick a 5 up [*points next to the 2*] there [*Ann puts ".5" next to 2 as answer*]. I meant to bring a calculator in today, but I forgot one. We'll have to bear without that. I'll help you on any of the math that we have to do. It looks to me that that's it, isn't it?

Okay, the reason I asked that now is that you were telling me this two and a half [*points to Ann's written answer, 2.5*] tells me how many forties will go into one hundred, right? [*Ann nods*.] So, you were saying originally, well this is two and a half forties [*points to Ann's scratchwork*] which means two times forty is eighty and a half of a forty is twenty, so that makes the one hundred. But doesn't … does this also tell you the amount of time?

Ann: Yeah.

73		Bill:	So, there is a direct connection, you see, between that number [*points to 40*] and that [*points to 2.5*].
75		Ann:	That [*points to her scratch work*] only tells the amount of time going over.
76		Bill:	Okay. I'll buy that.
77		Ann:	You still need to do 30 to go back.
78		Bill:	All right. Let's try that.
79		Ann:	[*Writes in long division form 100÷30=3*] And it would be 3 seconds back.
80		Bill:	Uh huh.
81		Ann:	Because 30 goes into 100, three times.
82		Bill:	And you'll have a little bit left over, I think.
83		Ann:	Yeah. [*Begins to actually divide, while explaining what she's doing*.] And then you have to put a another point because you can't go back in.
85		Bill:	You end up with another hundred there, you see. This is going to keep repeating, isn't it?
87		Ann:	Yes. It's gonna go … [*writes ".3333333333" next to 3*].
88		Bill:	Yeah. Okay? So, the total time here … let's just round them off because we're only going to one decimal place on the timer. How long do you think it's going to take him now to go over and back?
91	9:55	Ann:	3.3 [*draws a box around 3.3*] seconds plus [*writes 3.3+2.5 in column form*] … plus 2.5.
93		Bill:	Okay.
94		Ann:	Which [*writes answer, 5.8*] is 5.8 seconds.
95		Bill:	All right. Good. Now [*turns Ann's paper around to get a look at it*]. Let's run [*points to the computer screen*] him and then I'm going to come back and ask you not to look at this [*puts hand over Ann's paper*] and just tell me what this means in terms of the time thing [*moves hand over and back*] related to the speed again. Okay. Let's see if we're right, though [*gestures to the screen*]. I think you probably were. [*Ann activates turtle*]. You had 2.5 [*turtle comes back*]. It looks like it was right on. We'll have to ask Dr. Thompson why the happy face comes up on us. [*Ann laughs*.] Bingo! [*Turtle finishes*] Right on the money. Very good. If I showed the additional decimal points [*points to the Time Counter*] behind that, what do you think they'd be?
104		Ann:	8, 8, 8. I think they'd all be 8s probably.
105	9:56	Bill:	It's not really important, but I was just curious [*grabs the mouse to display more digits*]. I'll just tease you a bit here, you see. Remember all those 3s that were going to go on forever [*points to Ann's paper and drags finger across it*]?
108		Ann:	Yeah.
109		Bill:	We had all those. We didn't have a row of 5s, but we did have the 3s. Okay. Let's come back to where we were at [*puts the decimal places back to tenths*]. Now,

without looking at this [*turns paper upside down and looks at what is on that previously used side*]--pretty triangles and stuff--without looking at that now, explain to me the best you can what the … relationship is between the speed that we have here [*points to the Turtle-Over Box*], distance traveled [*moves fingers apart and holds them on the distance line*], and the time.

Ann: The relationship between the distance traveled and the time?

Bill: Yeah, well whatever speed he's running at [*points again to Turtle-Over Box*], and distance traveled whether it's 100 feet or 200 feet [*waves hand over and back a few times*], and the amount of time.

9:57 Ann: [*Uses the mouse as an on-screen pointer, but the camera does not allow us to see where she points*]. He has time here, okay? 40, and um … what that means is that he will go 40 … 40 feet per second, whi-which he will go every second as far as he goes over.

Bill: Uh huh.

Ann: And then he'll go 30 [*still using the mouse as a pointer*], which means 30 feet per second, when he goes back.

Bill: Okay.

Ann: Which means he would go 10 feet slower, so it would take him longer to do that.

Bill: "Ten feet slower." What does "10 feet slower" mean?

Ann: That means instead of doing … like, instead of doing 40 miles per hour,

Bill: Uh huh.

Ann: --if you were only doing 30 miles per hour, if you were going on a race with someone [*fidgets with the mouse*], the one who was going 40 miles per hour would win.

Bill: Hmmm! [*Nods*].

Ann: Because they're going faster then you were.

Bill: Okay.

Ann: And you would always be, if you kept up the same pace, you would always be 10 miles … behind him … or her.

Bill: What if we raced for 3 hours? How far behind him would I be? You're going at 40 miles per hour [*gestures with hands over and back*] and I'm going 30 miles per hour and we race for 3 hours [*Ann begins to fidget with pen*]. How far behind you will I be?

Ann: Ten miles. … If you have already crossed the finish line [*pretends to draw a finish line*].

Bill: Well lets say it's a … 24 hour race, okay? We're gonna race all day and all night. And you're going 40 miles per hour [*points to Ann*]. I'm going [*gestures to self*] 30 miles per hour. At the end of one hour [*gestures with hand to indicate the passage of time*], how far behind you will I be [*makes a space with fingers on desk for distance*]?

Ann: Ummm … 10 feet.

Bill: We're racing …

Ann: Ten miles.

Bill: [*Nods*] Ten miles, okay? Now we keep on racing. We don't stop. We just keep on going. Another hour goes by. How far will I be behind you?

Ann: Twenty miles?

Bill: Yeah [*nods*].

Ann: So, they would just add up?

Bill: Sure. Because every hour that we race, you're going 10 miles per hour faster than I am. So I get 10 miles further behind every time we go for another hour … Okay?

Ann: [*Nods*] Okay.

Bill: Now. Let's try going back to this again [*points to computer*]. I don't … I'm not too concerned about which way we go, but let's say … let's just use the rabbit now because he's going to go over and back and we don't have to set two different speeds here. Let's say we want the rabbit to go over and back in … 5 seconds.

9:59 Ann: Five seconds.

Bill: Uh huh.

Ann: Then, we would have to set him at 40 feet per second.

Bill: For what?

Ann: We'd have to set him [*points to the keyboard*] at 40 feet per second.

Bill: How do you know that?

Ann: Because we did that yesterday.

Bill: Ahh. Okay [*Ann chuckles*]. Let's use one we didn't do--you can't remember from yesterday. Umm … how about seven and a half seconds?

Ann: Seven and a half seconds?

Bill: Yeah.

Ann: Well … that would mean you could take less time. And … um … I'd estimate maybe it would be [*pauses, thinking*] 25 miles per hour or 20.5 miles per hour or something?

Bill: Oh, feet per second [*pointing to the computer screen*] you mean.

Ann: Yeah, feet per second.

10:00 Bill: It could be. Here. It could be. Let-let's go back a step

Bill: [*Brings out a new sheet of scratch paper.*] Let me just draw something right here [*draws a line segment*]. We're going to say that this is the 100 feet that's up there [*draws a tick mark at both ends of the segment; labels them "0" and "100"*], okay? And I'm not for the moment going to divide that up into any distance per se, but we'll just say this is 0 and this is 100. If we have the turtle or the rabbit running at … um … let's say 40 feet per second [*Ann nods*]. Down here we're going to have a graph of time [*draws a time line under the distance line*]. Okay? [*Ann nods*]. This is 0 seconds

[*draws a tick mark on the time line's left end*] and this [*draws tick mark on the time line's right end*] is whatever time it takes him to get down to the end [*points to "100" on the distance line*]. If he's running … let's say he is running at 40 feet per second [*writes "40 ft/sec"*]. Can you diagram on there [*points to the distance line*] where he's going to be at each second [*makes a space between his fingers on the distance line*] and where those seconds are on this graph [*points to the time line*] at the same time? Let me just show you what I mean. This ending point here [*highlights the right tick mark on the distance line*] is the same as the ending point here [*highlights the right tick mark on the time line*]. So when he reaches from here to the end [*moves pen from left to right on distance line*], he's gone from zero time to whatever that time is at the end [*moves pen from left to right on the time line*].

Ann: Is he going back [*indicates over and back*]?

Bill: No, let's just take him one way for the moment.

Ann: Just one way?

Bill: Uh huh.

Ann: Well, after 40, if he went 40 feet for one second … he would be … here [*highlights approximately one-third of the distance line*]

Bill: Okay.

Ann: … after 1 second [*writes "1 sec"*]. And that would be like here, right [*indicates approximately one-tenth of the time line*]?

Bill: That's what I want to see. Just go ahead and do it, okay?

Ann: And for the second second,

Bill: Uh huh.

Ann: … he would go the same 40 feet [*extends highlighted segment to about three-fourths of the distance line*]. So that would be [*writes "2 sec" alongside distance segment and draws a second tick mark, labeled "2", to indicate a total of approximately one-fifth of the time line*].

Bill: Okay. You want to label this up here, 40 feet and 40 feet so we know what we're talking about. [*Ann complies, writing "40 ft" and "40 ft"*] Okay, that's good. All righty.

Ann: Then you have [*draws a tick mark in the "left over" region of the distance line*] 2 … 20, 20 … 20 feet left [*draws a bracket over this region*].

Bill: Okay. [*Ann writes "20 ft" on top of bracket*]. And how long is it going to take him to do that?

Ann: It would take him half as much time as it takes this [*taps the end of the second 40 ft*]. So it would be half a second.

Bill: Okay [*nods*]. And where would that be down there [*gestures to the time line*]?

Ann: That would be like … over here [writes "1/2" above a tick mark a short distance from the 2 on the time line].

Bill: Okay.

Ann: So it would take him

Bill: [*Interrupting*] Remember what I was saying on this diagram down here [*time line*] that we want. This is the starting point [*points to the 0 on the time line*]. That's the ending point [*points to right end point of the time line*].

Ann: Ohhh [*draws a new time line below the old. Puts a 0 on the left and a 2 1/2 on the right*].

Bill: Okay, good.

Ann: That'd be one second [*draws a tick mark about a third of the way from the left and labels it "1"*], two second [*draws a tick mark about two thirds of the way from the left and labels it "2"*], and a half [*draws a smaller tick mark about a fifth of the way from the right and labels it "1/2"*].

Bill: Okay. Is this the two and a half mark [*points "1/2"*], or is that the two and a half mark [*points "2 1/2," written near the end of the distance line*]?

Ann: This one [*writes "1/2" over 2 1/2 at the end of the distance line and scribbles out previous 1/2*].

Bill: Okay. Good! Now, let's assume he's going to run at … um, some different speed. Why don't you do the same kind of thing on your own over there [*gestures to scratch paper*]. What if he's going to run at, ummm, 45?

10:03 Ann: [*Draws a new distance line*] Forty-five?

Bill: Yeah. Feet per second.

Ann: [*Draws a time line to the right and below this distance line, only two thirds of the other's length*] Okay, so if he's going he will go 5 more feet than, five more feet than up there. [*Mumbles something unintelligible, then makes a curved line between 0 and a point less than half way across the distance line*].

Bill: Okay. If you want, you can mark it like this instead of making the loops [*note: the curved lines to denote distance*], but it's up to you, either way.

Ann: He traveled … 45 feet [*writes "45 feet" above distance line*].

Bill: Okay. Where will he be on the time scale?

Ann: And that will be one second [*makes a tick mark about a fifth from the left and labels it "1 sec"* … Okay. This [*referring to the second second on the distance line*] would be … wait a second. 45 plus 25 is … Oh, I get it. This would be [*draws a second curved line from the end of the last one to a point just shy of the end point of the distance line*] …

10:04 Bill: Good show.

Ann: … 45 feet again [*labels the new curved line "45 feet"*].

Bill: Uh huh.

Ann: This [*referring to the time line*]would be … [*makes a tick mark about three quarters of the way from the left, labelling it "2 sec"*] here.

Bill: Okay.

Ann: That would be two seconds. And then this tiny bit right here [*points to the region on the far right of the distance line*] is what's left. And that would be … this is not, that would be 90 [*points to the first and second distance curves on the distance line*], so it would be 10 feet.

Bill: Good. That's right [*draws a final curved line and "10 feet left" on top of it*]. And, how long does it take him to go that 10 feet?

Ann: It would take him … Okay. A fourth of a second?

Bill: Pretty close [*Ann writes "2 1/4" at the end of the time line*]. How did you come up with a fourth?

Ann: It can't be a half and it's too small to be a third.

Bill: Yeah. Why can't it be a half? Tell me what you're thinking.

10:05 Ann: Half … Okay. Half of 45 [*writes "45",*]

Bill: Oh, okay, I understand.

Ann: is not …

Bill: That would be twenty-two and a half.

Ann: [*Writes "22 1/2" below the 45*] Yeah.

Bill: Okay, so it's not that, is it?

Ann: And half of this [*points to 22 1/2*]

Bill: Uh huh.

Ann: is less. Half of

Bill: Yeah, it's about eleven and a quarter. [*Ann writes "11 1/4"*] And he's got to go ten feet.

Ann: He only has to go 10 feet so you just use the quarter from here [*circles the 1/4 of 11 1/4*].

Bill: Okay, so it's a little less than a quarter. We won't dwell on that for the moment. But here is what I want to come back to. Do you see the distance you have here from zero to one second and one to two seconds [*drags finger along the time line*]?

Ann: Uh huh.

Bill: Should those be the same distances?

Ann: No.

Bill: Isn't one second as long as the first and second second?

Ann: Yeah. But it can't be perfect.

Bill: I'm not saying about your drawing, I'm just saying in reality, though.

Ann: Yeah.

Bill: If we were graphing time, would we have equal … ?

Ann: [*Interrupting*] If you were like a scientist or something and you were graphing time, you would make sure that they were like even. You'd probably have a ruler or something [*pretends to measure the time line with a ruler*].

Bill: Hmm. Okay [*nods*]. I want to make sure you understand, though, that one second is the same length as the next second and the next second.

[*moves hands to indicate second intervals in the air*] unless they're variable [*Ann nods*], and I don't know about that [*Ann mumbles something unintelligible*] Okay. Put this one aside and grab another piece of paper. Question for you. If you know, let me use your pen for a second. If you know now that that same distance here [*draws a distance line with out tick marks save ones at the ends*] is 100 feet and the graph down here for the time [*does the same thing for time, making them both the same length*], I'll make it the same length down here just to make it a little easier, and we know that the ending down here is seven and a half seconds [*writes "7 1/2 [7.5]" at right end of the time distance line*], or 7.5 seconds, which ever you want to call it. What speed does he have to go to so that this [*points to time line*] matches that [*points to the distance line*]?

10:07 Ann: [*Writes a "0" at the left of both lines, and "100 ft" at the right end of the time distance line*] Okay, seven and a half seconds.

Bill: Uh huh.

Ann: [*Pauses, looking quizzically at the two lines. She then looks at Bill.*] Half of 7 is 3.5, right?

Bill: Okay [*nods*].

Ann: [*Pause*] So … that [*draws an arrow from the 7 1/2 pointing down. Where this arrow points, Ann writes 3 1/2 + 3 1/2 = in column form*].

Bill: So, where on this time line would that be [*gestures across the time line*], this three and a half? On this one [*points again to the time line*].

Ann: That's not important.

Bill: O-okay.

Ann: Half of seven is three and a half.

Bill: I'm sorry, it's three and three quarters. My apologies, I shouldn't have misled you there [*Ann changes addition column to read 3 1/4 + 3 1/4*]. Three and three quarters. Three over four. Or 3.75 and 3.75. If you want to use an easier one, just use straight 7, that's all right.

Ann: [*Crosses out addition column by drawing a triangle over it. She pauses, then writes "7 sec." near the middle of the page*] Okay.

Bill: Okay. We're gonna use seven, all right?

10:08 Ann: It has to be 7 seconds [*draws a "x" [multiplication symbol] next to and slightly about the 7*].

Bill: Okay.

Ann: And if you have that times by, let's say a hundred maybe [*writes "100" above the 7 sec.*]? Because it's the distance of feet [*points to the right end of the distance line*] and this is the time that you want [*points to the 7 sec., then writes "feet" next to the hundred*] . So you times seven [*draws a line under 7 sec. and then "700" below that*] … And you get this [*note: the 700*] and you take it over [*draws an arrow to the right of the 700, rewrites 700, and puts a minus sign below it*]. And then you subtract, and then you subtract it by, umm [*pause*], by another number and you get the answer, I think [*chuckles*].

Bill: Okay. What does the 700 represent that you've done here?

Ann: It represents … 7 times 100.

Bill: Okay. But this [*points to 100 feet*] is number of feet, right?

Ann: Yeah.

Bill: And so you're saying this is 100 feet times 7 seconds. That's as if he's going 100 feet per second for seven seconds [*points to the 700*].

10:09 Ann: [*Looks down at her work for a moment*] Yeah [*looks up at Bill*].

Bill: He's not doing that, is he? He's not traveling 100 feet per second, is he?

Ann: [*Sounding sure of herself:*] That's why you have to subtract it [*taps the 700 with the pen*] by something [*smiles*]!

Bill: Ah. Okay. Let me back you up a minute. When you said here a minute ago [*touches the 0 and the 7 on the time line*] that if we're going to go for 7 seconds down to the end half of that 7 seconds, how far will he have gone in this 100 feet [*touches the 0 and the 100 on the distance line*]?

Ann: … Half way.

Bill: Put your mark down there [*gestures to the two lines; Ann marks the halfway points of both*]. How long will it take him to get half way?

Ann: [*Pause*] Three and a half seconds.

Bill: Uh huh [yes]. That's right. So from that is there a way that we can use this [*puts fingers at ends of time line*] to determine the speed [*indicates a part of distance line*] it's going to require him to get up there [*drags a finger along the distance line*]?

Ann: [*Softly*] No.

Bill: Remember the first thing we were working on this morning?

Ann: Uh huh [yes].

Bill: He was running at 40 feet per second. What did you do with the 40 feet per second to get the time it took him to go the full length … [*gestures across distance line*] or the 100 feet [*gesturing across distance line again*]?

Ann: … I divided it.

Bill: [*Nods*] Uh huh [yes]. Why?

Ann: So I could get the answer.

Bill: Okay, but when you divided the 40 into the 100 feet, in effect you were saying that gives me a 40 foot section here [*points to a first part of the distance line*], a 40 foot section here [*points further to the right on the distance line*], and I had 20 left over [*points to a portion at the end of the distance line*], right [*Ann nods*]? So that gave me a second, a second, and a half second.

Ann: Uh huh [yes].

Bill: What would prevent you from doing the same thing with the seconds?

Ann: [*Pauses while looking at her scratch paper*] I don't quite understand [*shakes head*].

Bill: Okay. Now, instead of knowing the speed [*holds thumb and index finger apart*], we know the time. I'm going to travel from here to there [*moves hand from 0 to 100 on Ann's distance line*] in 7 seconds. Okay?

Ann: Okay.

Bill: If I do that, how far … can you show me on here [*points to time line*], kind of generally speaking … if I do it in seven seconds, how far … you've marked here how far I've traveled in three and a half seconds [*points to the midpoint of both lines*], how far on that thing [*points to distance line*] would I have traveled in one second? Or two seconds? Or seven seconds?

Ann: [*Looks down at the paper. Sounding discouraged.*] The whole thing?

Bill: Seven seconds would be the whole thing, okay?[*points to the 7 on the time line*] How about one second, two seconds? Can you just make marks on there like you're going to put this into the sections showing how far you would go each second [*uses thumb and index finger to indicate succesive intervals*]? [*Ann puts five tick marks on the time line, haphazardly dividing it into seven intervals*] Okay. And you have [*counts the intervals*] 1-2-3-4-5-6-7 sections. Right? Do those correspond to sections up here [*indicates sections on Ann's distance line*]?

Ann: Yeah. [*Pause*] Yeah [*nods*].

Bill: [*Nods*] Okay, they do. Now the main thing we're trying to figure out is … what is that distance that he traveled in this one second [*drags finger over a small area of the distance line*]. How can I determine that from what you know now?

Ann: [*Looks at the paper for a long time*] I'm not sure.

Bill: Well, let's see. You guys were working with sharing between containers.

Ann: [*Looks up a Bill*] Uh huh.

Bill: [*Places hand over both lines*] In this case, let's say that each one of those containers, we've got seven of them, was one second. Okay? I want to share that 100 feet equally between seven seconds [*Ann pushes paper around on desk with pen while looking at Bill*]. How do I figure out how much each second can will get in terms of feet? [*Looks down at the paper, then vacantly, then back to paper for a very long pause*] Tell me what you're thinking, Ann. I don't want to get you stuck. If you get stuck on it, just say so and we'll take a different tack. But in the meantime let me know what you're thinking so I can figure out how to guide you to it.

10:14 Ann: I don't know [*chuckles*].

Bill: Okay, do you understand what we're talking about here [*touches the time distance line*] in terms of representing the time span [*Ann crosses out the "1/2 [7.5]" at the end of the time line while Bill talks*] along side the distance span [*points to the distance line*] with a hundred feet?

Ann: Yeah.

10:15 Bill: Okay. And the idea that we're expressing here [*grabs the pen*] was that if we go a certain distance [*moves the pen across the first time tick interval*] in one second, we're going at a constant speed now, okay? As we do that, we're going to go at the same distance in one second [*moves pen up to match the exact same interval space on the distance line*] between zero and one as we do from one to two [*drags pen further along the distance line to about halfway*] so that our time spans here [*darkens the "0" and next tick mark on the time line*] are directly related to the distance span. So if I can come down here [*moves hand to the time line*] and divide this line from zero to seven [*darkens the tick marks representing 0 and 7*] in seven [*touches each time tick interval*] equal second segments, each one of these segments would represent one second and there are seven of them there. Up here [*touches the distance line*], I'd be going along a similar line [*divides the distance line into corresponding tick intervals*], but this one isn't seconds, this one is divided into feet, or is in feet I should say. So from that, can I determine how far [*darkens the first distance line tick interval*] that distance is that I'm traveling in one second? How many second sections are there on this line [*makes marks underneath the next two tick intervals on the distance line*] going along with the ones down here [*touches the time line*]. I misled you there, I think. If I travel one second here [*darkens the one second mark on the time line*], how far in time have I traveled up here [*darkens the corresponding distance line tick mark*]?

10:16 Ann: [*Shrugs, then says softly:*] One second?

Bill: Yeah [*nods*]! In time, I've traveled one second up here too. When I get down to the end, I've traveled seven seconds [*darkens the end tick mark on the time line*]. How far in time [*darkens the end tick mark on the distance line*] have I traveled to there?

Ann: [*Softly:*] Seven seconds.

Bill: Okay. So, how do I figure out how far this is [*touching the first tick interval on the distance line*]? Just this one seconds worth of length up here [*again touching the same tick interval*]?

Ann: [*Pause. Then softly:*] I don't know.

Bill: [*Pauses.*] How many sections do we have up here [*touching the distance line*]?

Ann: Seven.

Bill: Are they all the same length?

Ann: Yeah.

Bill: [*Nods*] Yeah. How long is one of them?

Ann: One second long?

459		Bill:	One second long, but in feet? [*touches the first interval on the distance line*]
460		Ann:	[*Shrugs*] I don't know.
461		Bill:	[*Pause*] Let's not use something like seven. Let's say we have 100 feet [*makes a new smaller distance line labeling it at the end point "100"*] and now I'm going to be traveling at 25 feet per second [*makes one tick interval and labels it "25"*]. Right? How many sections would this be divided into then? [*Pause*] 25-50-75-100.
465		Ann:	Four.
466		Bill:	Right [*divides line up into four tick intervals and puts"4" under 100 at the end*]. How long will it take him to get down there?
468		Ann:	Four seconds?
469		Bill:	Good. I've got one that's five seconds [*makes another distance line below the 25 one, the same length, and labels it "5" at the end*]. How many sections will this [*drags pen over the new distance line*] be divided into?
472		Ann:	[*Pause*] Five?
473	10:18	Bill:	[*Nods*] And, if I do it into five [*marks off the distance line into five tick intervals*], how long is each one?
475		Ann:	Forty seconds? Forty miles per hour?
476		Bill:	Let's see if that makes sense. [*Puts pen on each interval in succession:*] Forty plus 40 plus 40 plus 40 plus 40. Remember, this is the 100 feet we're talking about here [*puts "100" over the 5 on the new distance line*].
479		Ann:	[*Pause*] Twenty?
480		Bill:	Yeah. How far did he travel in this time [*points to the first interval. Writes "20" over it*]?
482		Ann:	Twenty.
483		Bill:	How long did it take him to travel there?
484		Ann:	… One second?
485	10:19	Bill:	Good. Up here [*points to the distance line right above the new one, 100 in 4 seconds*], it was 25 and one second. Okay. What would it be there [*points to original distance line, 100 in 7 seconds*]? [*Ann looks down at the distance line, pauses, then shrugs*]
488		Bill:	You don't have to come up with the number, but how would you calculate the number? [*Portion of transcript omitted.*] This, we said, was going to take him four seconds, this was five seconds, do you see any relationship between this number [*4*] and this number [*100*] that would lead to that [*25*]? This number [*5*] and this number [*100*] that would lead to that [*20*]?
493		Ann:	[*Shrugs, then shakes her head.*] No.
494		Bill:	What about these two [*points to 100 and 4*]. What if I divide or multiply or add or subtract these two? Do I come up with that in any way?
496		Ann:	[*Softly.*] I don't know.

497		Bill:	Think about it for a second because that's the key right now. You're right on the verge of knowing the answer. [*Long pause*.] What do you think?
499		Ann:	I'm not sure.
500		Bill:	Well, tell me if you can see any relationships between these two numbers [*points to 100 and 4*]. This is four seconds. That's how long it took him to go 100 feet. Okay? … How can I end up with that [*points to 25*] as a speed?
503		Ann:	[*Long pause*.] By subtracting?
504	10:21	Bill:	[*Looking down at the scratch paper*] I don't see how--Show me how you would do that. How would you subtract it? [*Ann shrugs*] … Okay. Let me have you do this. Why don't you make a couple of columns here. And we're going to make the column here, how far he has traveled, and the time. Okay? So on the first one over here, let's say he's going to be traveling 25 miles per hour. [*Ann makes misshapen columns with "25" at the top*]. Okay? How far does he travel? Let's put one second, two seconds, three seconds, four seconds, whatever. How far does he travel in the first second?
511		Ann:	[*Softly*] Twenty-five miles.
512		Bill:	Okay, so let's record here the time and here the distance he's traveled. Okay. How far has he traveled in the second second.
514		Ann:	Fifty.
515		Bill:	Okay, let's record that. And the third and fourth, etc.
516		Ann:	Until 100?
517	10:22	Bill:	The fourth is 100? [*Ann writes "1 sec," "2 sec," "3 sec," "4 sec" on top of each other and below the 25. She uses a distance line to the right and parallel to the column to denote the distance per second travelled*] Okay. How far will he have traveled in … ten seconds? [*Ann begins to draw an extension to her newest distance line to make it 10 seconds long. She gives up and crosses the addition off, then writes "10" with possibly a minus sign, then crosses that out*.] Let's go back up here. Okay [*points to Ann's lines and times in column form*]. In one second, he went 25. Two you said he went 50, right? How did I get that?
525	10:23	Ann:	You added 25 plus 25.
526		Bill:	Oh, okay. And how about three seconds?
527		Ann:	You added another 25.
528		Bill:	Okay. Is there a more expedient, or easier way to add then just add 25 plus 25 plus 25?
530		Ann:	Yeah.
531		Bill:	What?
532		Ann:	Multiply.
533		Bill:	Sure. So if we get down here to 10 seconds, what are we going to do with the 10?
534		Ann:	… Multiply?

Bill: Sure! That's what you've been doing here, right?

Ann: [*Softly*] Yeah.

Bill: So, how far will he have traveled in 10? [*Ann writes 25 * 10 = in column form*].

10:24 Bill: I should have brought in the calculator. I apologize for that. I didn't bring one. There you go. That's correct, it would be 250 [*Ann writes "250" under her multiplication column*]. Okay?

Ann: Okay.

Bill: Now. You see how we were going down here and just taking the time and the speed and you were multiplying the time [*points to 1 sec. from Ann's columns*] times the speed [*points to the 25 on the top of one of the columns*] to get the distance?

Ann: Okay.

Bill: Okay. Is there a way of seeing a relationship between these [*points to 4 and 100 on the top small distance line*] and this [*points to the 25 by the same distance line*]. From that what you were just doing, what were we doing to get from here to here to here, etc. [*points at each of the tick marks on the top small distance line in reverse order*]?

Ann: We were … [*pause, shrugs*] I don't know.

Bill: Well, what were you doing to get from this column [*points to the seconds column*] to this column [*points to the distance line displayed vertically by the seconds column*]? [*Pauses, waiting for Ann to respond*] You were multiplying weren't you?

Ann: … Yeah [*shrugs and then nods*].

10:25 Bill: Yeah. Okay. So if we're going down this track … [*points to the top small distance line*] in segments [*touches the first tick interval, labelled 25*], we happen to know now are 25 feet per second each time, we're going [*drags finger over the distance line*] to get to the 100 in 4 seconds. Four times the 25 is 100, right?

Ann: [*Softly*] Yeah.

Bill: Okay. What's the relationship between the 5, 100 and 20 [*touches these numbers on the bottom small distance line*] … on this one [*taps the bottom distance line again*]. If we're going 20 feet per second?

Ann: … [*Softly*] They all make up the problem?

Bill: Huh?

Ann: They all make up the problem?

Bill: Well, yeah, that's a different problem. But what I'm asking you is to see if there's a relationship between these three numbers [*points to the 4, 25, and 100 on the top distance line*]? … Okay. Is there a relationship between 4, 25, and 100?

Ann: No?

Bill: Okay. How about here [*touches the columned numbers in succession*] 25, one, two was 50, three was 75, four was a hundred. Here's the same ones I'm talking about right here. [*Pauses, waiting for Ann to respond. Ann looks discouraged*]. Is there any

way I can get 100 from knowing these two numbers [*points to the 4 and the 25 on the top distance line*]?

10:26 Ann: [*Shrugs*] Umm [*long pause*]. Yeah.

Bill: How?

Ann: I'm stuck.

Bill: Okay. What did we just do here, Ann? You did it, I didn't do it.

Ann: [*Softly*] We multiplied.

Bill: Okay. Is there anyway of getting to 100 knowing these two numbers [*again pointing to the 4 and the 25*]?

Ann: Uhh … Yeah.

Bill: How?

Ann: Multiply?

Bill: If you multiply, do you get 4 times 25 is 100?

Ann: [*Softly*] Yeah.

Bill: Sure you do. I guess our bell just rang. Ann, don't get so worried about it. You're doing fine. But there's an answer here that I can't give you. You gotta see it for yourself. And when you do, it will make your light bulb go pop! So don't worry about it. You're doing just fine. [*Ann nods*]. See you tomorrow. With any sort of luck, I'll be here.