00:00:00 to 00:01:58

1. Pat: This is for you [*places a calculator in front of Ann*].
2. Ann: Why does it always have that smiley face on it [*note: the computer screen]*?
3. Pat: Yeah, I thought that you might ask me about that. There's a really obscure reason for that. And it has to do with the programming language that I used to write that program. And it's doing … now don't laugh when I tell you this: it's collecting garbage.
4. Ann: So it smiles to collect garbage?
5. Pat: That's right. It smiles to collect garbage.
6. Ann: Okay.
7. Pat: What that means is it's cleaning up the computer's memory because as the program runs, it leaves little fragments of stuff in the computer's memory and then when it runs out of useable memory it goes back and sweeps [*waves arm back and forth*] out all those unusable fragments that it made. [*Ann nods*]. So that's why it's called garbage collecting.
8. Ann: You should see how fast this rabbit can go.
9. Pat: How fast have you made it go?
10. Ann: Nine, nine, nine, nine, nine, nine, nine, nine, …
11. Pat: I bet you it just went pssst! [*Waves finger over and back quickly*].
12. Ann: Watch.
13. Pat: All right.
14. Ann: [*Types "9999999" for the Rabbit-speed Box*] Then you make it run.
15. Pat: All right.
16. Ann: It's funny, watch! [*Activates the rabbit*] It doesn't take any time at all.
17. Pat: No, I just saw a little flash over there [*points to the right end of the distance line, at 100 ft*], didn't you? Wait . Here [*takes the mouse*]. Did I take no time at all?
18. Ann: Yeah.
19. Pat: [*Changes the Over and Back controls to show more decimal places on the Time Counter*] Or it just didn't … it took too little time to …
20. Ann: … to record maybe?
21. Pat: Suppose it took a millionth of a second. Could we see it here [*points to the Time Counter*]?

22. Ann: [*Reactivates the rabbit*] See, it still didn't take anything [*meaning that even with the extra decimal places the Time Counter still showed 0*].

23. Pat: What's the smallest amount of time that it could register [*taps on the computer screen at the Time Counter*]?

24. Ann: Umm … a thousandth.

25. Pat: A thousandth of a second. So if it took a millionth of a second to go [*Ann types "35" for the Rabbit-speed Box*] over and back … [*moves pen over and back in the air a few times*].

26. Ann: You wouldn't see it.

27. Pat: Well, it wouldn't register in the time. [*Ann activates the race, with the Turtle-Over Box at 30 and the Turtle-Back Box at 40. The rabbit wins*]

Excerpt 1 — 00:01:58 to 00:03:41

1. Pat: Did you have some confusion yesterday?

2. Ann: Yeah.

3. Pat: A little bit, huh? Well instead of talking about turtle and rabbit right now, let me ask you some questions about time, speed, and distance, okay?

4. Ann: Okay.

5. Pat: Let's try and clear things up [*draws a distance line with tick marks as end points*]. Now yesterday Mr. B drew something like this, right?

6. Ann: Yeah.

7. Pat: And gave that a distance. I'm going to give it a really messy distance, like five hundred and twenty-three feet [*writes "523" at the right end of the distance line*]. Okay? Now if we cut this up into [*puts four equally-spaced tick marks between the end points*] … one, two, three, four, five parts, what would you do to find out how long one of these parts was?

8. Ann: … I would …

9. Pat: I'm not asking for an answer, I'm just asking what you would do.

10. Ann: Divide five into five hundred and twenty-three?

11. Pat: All right. So one of these parts [*writes "523 ÷ 5"; draws a line from it to the rightmost interval*] would be five hundred and twenty-three divided by five. Okay? [*Ann nods*] So that's distance [*writes "ft" next to the "523"*]. I'm calling that feet. Remember I'm not asking for an answer.

12. Ann: Uh huh [*yes*].

13. Pat: That [*points to "523÷5"*] would tell you how long just this one is? [*points to the last interval again*]
14. Ann: Uh huh [*yes*].
15. Pat: Or would it tell you how long each of them is? [*points to each interval individually*]
16. Ann: It would tell you how long one of them was.
17. Pat: And how would you find out how long this one was, right there … the second one? [*draws brackets under the second interval from right*]
18. Ann: You … if you wanted to find out just these two? [*points to the two rightmost intervals*]
19. Pat: No, not how much they are together, how much they are individually [*again points to all the intervals from right to left*].
20. Ann: They would all be the same!

00:03:43 to 00:04:39

Pat checks to see that Ann understands that all these pieces put back together gives the total distance. At first she thinks he's asking only for the next four pieces, but clears up the confusion.

1. Pat: Oh, they're all the same. So when you calculate this one, the length of this piece [*draws brackets under the far right tick interval*].
2. Ann: Yeah.
3. Pat: You'd get the length of each piece.
4. Ann: [*Appears distracted by something behind Pat*] Uh huh [*nods*].
5. Pat: Is that correct?
6. Ann: Yeah.
7. Pat: All right.
8. Ann: But if you wanted the length of the whole thing [*drags finger all the way across the distance line*] you would have to, umm, times, multiply.
9. Pat: All right, so you'd multiply this number right here [*points to the last tick interval, labeled 523÷5*] …
10. Ann: Yeah.
11. Pat: … by what?
12. Ann: By four, so you'd get the rest.
13. Pat: Four?

14. Ann: Yeah.

15. Pat: Oh, to get how much these four all are together [*points at each of the first four tick intervals*]?

16. Ann: Yeah.

17. Pat: What if you wanted to get how much all five are [*drags pencil across distance line*] altogether?

18. Ann: Times five.

19. Pat: And what would you get?

20. Ann: … Your answer. [*Pat continues to stare at Ann. She chuckles*] Five hundred and twenty-three.

21. Pat: You'd get the whole thing wouldn't you? [*Ann nods*] Okay. So divide five hundred twenty-three by five to get the length of one of them [*points to the last tick interval*] and then you're right, you multiply how many pieces you have [*points to each tick interval from right to left*] by that number [*points to the 523÷5*] to get the length of how many pieces you've got [*again pointing to each tick interval*]. Okay?

Excerpt 2 — 00:04:39 to 00:06:01

1. Pat: Now down here … okay, let's forget distance for a little while. Suppose that we're running the rabbit just one way [*moves hand across the computer screen to indicate Rabbit's trip over*]. And I'm going to use a segment here to talk about how much time it takes [*draws a smaller line below the distance line, with tick marks as end points*]. Now here's the way I want you to think about this, because I'm not talking about distance with this line [*points to time line*]. I'm just talking about time. As the rabbit goes over … now watch my finger [*puts his left index finger at the beginning of the distance line*]. As the rabbit goes over here [*slowly moves index finger across the distance line*], he's going to … what's this timer doing? [*points to the computer's Time Counter*]

2. Ann: It's timing how long it's taking him

3. Pat: Okay.

4. Ann: … to go from one place to another.

5. Pat: So the number of seconds … what's happening to the number of seconds as he goes? [*moves finger slowly across the distance line*]

6. Ann: They're increasing.

7. Pat: It's increasing. Okay. So as he goes we can also think, if this is seconds … [*writes "seconds" at the right end of the time line*] … As he goes along a distance line [*drags finger across the distance line*], we can think of the number of seconds increasing also [*moves index finger on the right hand across the time line*]. We can think of them doing it together [*moves fingers across both lines simultaneously*].

8. Ann: … Yeah.

00:06:01 to 00:07:06

1. Pat: Okay? So, if we ask … When he gets out here [*spreads thumb and forefinger apart over first distance interval*], he's gone what part of the whole distance?
2. Ann: One-fifth.
3. Pat: One-fifth. Now when he goes one-fifth [*drags finger to the end of the first interval on the distance line*] … now remember, as he's going, the time is increasing too [*Ann nods*]. So as he goes along … suppose that we freeze everything as soon as he hits that one-fifth spot [*drags finger again to the first tick mark*]. And let's say that this is the total amount of time that he takes [*puts finger over the entire time line*]. We don't know how many seconds there are, but that's just the total amount of time.
4. Ann: So it would be a fifth of the time, of the total.
5. Pat: Very good! Yeah. If we freeze him here [*drags finger across the distance line to the first tick mark*] at one-fifth of the way along the distance that he's going to go [*places fingers along the entire distance line*], then the time marker is frozen where? [*points to the time line*]
6. Ann: The same distance? Like the fifth?
7. Pat: Now remember these are seconds [*pointing to the time line*], these aren't feet. Oh, I'm sorry [*pats Ann on the shoulder*], go ahead.
8. Ann: Like a fifth of the whole thing [*nodding to the time line*].
9. Pat: Okay. A fifth of the whole number of seconds [*Ann nods slightly*].

00:07:06 to 00:08:54

Pat asks Ann to suppose it takes him 2 seconds to go 1/5 of the distance. Then asks how long it will take to go 2/5 of the distance, 3/5 of the distance. Then asks her to drop the 2 seconds. Ann then says that it will take the rabbit 3/5 of the total time.

1. Pat: All right, let's suppose, all right, that as he goes out here [*again dragging finger along the distance line to the first tick mark*] and we freeze it at one-fifth of the way, so it's frozen one-fifth of the way in the time also. Let's suppose that it took him … two seconds [*makes a tick mark on the time line about a fifth of the way, and writes "2" beneath it*]. That one-fifth of the distance [*drags finger along the distance line to the first tick mark*], we get one-fifth of the time, and that's two seconds. Can you tell me anything about how long it's going to take him to go?
2. Ann: Well if it takes him two seconds to get over one-fifth, … um, since there are five parts [*moves hand over the distance line*], and you would just times five [*points to the distance line*] by two [*points to the first tick mark on the time line*].

3. Pat: Okay [*nods*]. Very good. That's very good. Now if we take him over here and we freeze him [*drags finger across the distance line to the second tick mark*] at two-fifths of the way, then how far would we be over here [*holds pen over the time line*] in the number of seconds that it's going to take him?

4. Ann: Four seconds.

5. Pat: Four seconds [*makes a second time mark on the time line*]. Okay. So this is one-fifth of the time [*writes "1/5 of the time" under the first tick interval on the time line*]. This is one-fifth of the distance [*writes "1/5 of the distance" under the first tick interval on the distance line*]. Suppose that we take him out here [*drags finger to third tick mark on the distance line*] at three-fifths of the way along the distance and we freeze him. Then how far along on the time [*drags pen over the time line*] would we be?

6. Ann: Uh, six … seconds?

7. Pat: Okay, six seconds, if it takes him two seconds to go one fifth of the way.

8. Ann: Yeah [*nods*].

9. Pat: Let's drop the two seconds [*scratches out the 2 beneath the first tick mark on the time line*] for now. If we go three-fifths of the distance [*drags finger to the third tick mark on the distance line*], what about the time [*gestures over on the time line about halfway*]?

10. Ann: It'll be three-fifths of the … total number of seconds [*drags finger across the time line*] that it would take him.

Excerpt 3 — 00:08:54 to 00:09:05

1. Pat: Okay, yeah. And we don't even need to know the number of seconds do we? [*Ann shakes head*]. It's just three-fifths of the distance [*drags finger across the distance line to the third tick mark*] goes with three-fifths of the time [*drags pen across the time line to about halfway*]. What about all the distance? [*drags finger all the way across the distance line*]

2. Ann: That would be all the time.

3. Pat: All the time.

00:09:07 to 00:11:00

Pat uses the computer. Sets Rabbit's speed at 43 ft/sec. P: What does that mean? A: He goes 43 feet every one second. P: First second only? A: First second and second second and the third second and however long it takes him. Pat changes speed to 60 ft/sec. P: How far will he go in 2 1/2 seconds? A: 150 ft. P: How did you figure that? A: Because 60 plus 60 is 120 and then half of 60 is 150. P: And what were those sixties? A: No. of feet … per second. P: Instead of saying "per second" let's tie these two together. The first sixty was how far he went in the first second. The other sixty was how far he went in the second second. And then the 30 was … A: How far he went in half of the next second.

1. Pat: Okay. Now … Now I'm going to just approximate. [*Types "43" for the Rabbit-speed Box*] Let's say the rabbit's going forty-three feet per second. What is that [*points at the Rabbit-speed Box*] going to mean?

2. Ann: It's going to mean that he goes forty-three feet every one second.

3. Pat: Every one second [*nods*]. Okay. Is that just the first second?

4. Ann: No, it's the first second and the second second and the third second [*displays fingers to show each second she is talking about*], and however long it takes him [*Pat nods*].

5. Pat: All right, so each second he'll go forty-three feet, right? How far will he go in two and a half seconds? I'm sorry, that's a crazy number for two and a half seconds. [*Changes the Rabbit-speed Box to "60"*] Let's say sixty feet per second. How far will he go in two and a half seconds?

6. Ann: He would go … a hundred and fifty seconds.

7. Pat: How did you figure that? You figured that very quickly.

8. Ann: Because sixty plus sixty is a hundred and twenty.

9. Pat: Okay [*nods*]. And sixty …

10. Ann: And half of sixty is thirty.

11. Pat: And what were those sixties that you were adding?

12. Ann: The sixties were the number of feet, or …

13. Pat: The number of feet … ? [*Waits for a response*].

14. Ann: Per second.

15. Pat: Okay. Instead of saying per second, let's tie these two together, like [*points to the distance line and the time line simultaneously*]. The sixty [*points to the Rabbit-speed Box*] is how far he went in one second [*points to the first tick interval on the distance line, then touches the first tick mark on the time line*].

16. Ann: Uh huh [*nods*].

17. Pat: The other sixty is how far he went in the second second [*points to the second tick interval on the distance line, then touches the second tick mark on the time line*] …

18. Ann: Uh huh.

19. Pat: … and the thirty was [*touches the middle of the third tick interval on the distance line*] …

20. Ann: Half of the next second.

21. Pat: Okay. So if we were to say like we did before, the first sixty [*touches the first tick interval on the distance line*] was how far he went in the first second [*points to the first*

tick mark on the time line], the second sixty [*touches the second tick interval on the distance line*] was how far he went in the second second [*points to the second tick mark on the time line*] and so the thirty [*touches a point halfway along the third tick interval on the distance line*] is how far [*points to the halfway point on the time line*] he went … [*waiting for a response*]

22. Ann: In half of the … half of the next second.

23. Pat: Yeah [*nods*], the next half.

24. Ann: Yeah.

Excerpt 4 — 00:11:02 to 00:12:10

1. Pat: Now. Suppose I turn that around. We've still got distance and we've still got time, okay? … [*On a new piece of paper, draws two lines like that on the previous page, the bottom one shorter than the other*] We're going to do … another one. Now you understand, still, that even though I've got a length here [*points to the bottom line*], I'm talking about time.

2. Ann: Yeah.

3. Pat: All right [*writes "Time" next to the bottom line*]. And that's just all the time it's going to take him.

4. Ann: Uh huh [*yes*].

5. Pat: And this is all the distance [*writes "Distance" beside the top line*] that he's going to go. And let's suppose that this is one hundred feet that he's going to go [*writes "100" at the right end of this new distance line*]. Suppose that we say, "All right, let's cut up the time that he's going to go." … [*Interrupting himself.*] When he uses up all the time [*drags the pen across the time line*] where is he going to be?

6. Ann: Over and back?

7. Pat: Let's just talk about him going one way [*holds up one finger*].

8. Ann: Okay. He'd be all the way … a hundred feet.

9. Pat: A hundred feet. So when he uses up all that time [*gestures across the time line*], do we have to know how much time there is [*again gestures across the time line*] to say

10. Ann: No [*shakes head*].

11. Pat: So you just know that when he uses up all that time [*gestures across the time line*] he's going to go a hundred feet [*gestures across the distance line*].

12. Ann: Uh huh [*nods yes*]

Excerpt 5 — 00:12:10 to 00:13:37

1. Pat: Okay? Suppose that I tell you that when he uses up half his time [*puts pen at the halfway point on the time line*; *looks up at the computer screen; pauses*] … No, I'm getting confused on this. [*Pause.*] When he uses up half his time … where will he be when he uses up half his time?
2. Ann: Fifty feet.
3. Pat: Fifty feet. Where will he be when he uses up one-fourth of his time [*places pen about a fourth of the way along the time line*]?
4. Ann: [*Pause*] Umm … twenty feet?
5. Pat: Okay. And how are you coming up with twenty?
6. Ann: If half of a hundred is fifty then, … umm, like a fourth of a hundred is like twenty or thirty.
7. Pat: [*Pause.*] I'll tell you twenty is wrong. But if you had told me that it's a fourth of a hundred, that's right. So if you're not sure, like, what a fourth of a hundred is, just say a fourth of a hundred [*shrugs shoulders. Ann nods*]. Because we can always use a calculator to get a number, right?
8. Ann: Yeah.
9. Pat: Okay. So if he goes a fourth of the way in the time [*points to about a quarter of the way across the time line*], he's going to go how far in the distance?
10. Ann: A fourth of the way.
11. Pat: [*Nods*] A fourth of the way. So a fourth of one hundred. Okay. And what would you [*points at Ann*] do with the calculator to find out what a fourth of one hundred is?
12. Ann: Umm, you divide … four into a hundred?

Excerpt 6 — 00:13:37 to 00:15:12

1. Pat: … Yes [*nods*]. That's right … A seventh of the way, he would be how far on the distance?
2. Ann: A seventh of the way.
3. Pat: A seventh of … ?
4. Ann: He'd be a seventh of a hundred.
5. Pat: A seventh of a hundred. All right. Suppose that I tell you that it takes him [*writes "7" at the end of the time line*] seven seconds to go a hundred feet [*gestures across the distance line*]. Now, let's not worry about speed, okay? Let's just talk about … so if it takes him seven seconds [*gestures across the time line*] to go across the time, how many parts [*pretends to count tick marks on the time line*] have we cut up the time into?

6. Ann: … Seven.

7. Pat: Okay. So [*divides the time line with the pen into seven intervals, and draws a squiggle beneath the first one*] … And then each second is what part of the total time [*gestures toward the time line*]?

8. Ann: One-seventh.

9. Pat: One-seventh. So that's one-seventh of the time [*writes "1/7 of the time"*]. How much would one-seventh of the time go with up here [*points to the distance line*]?

10. Ann: One-seventh of the distance.

11. Pat: One seventh of the distance. Okay. So one second, this is one second [*writes "1 sec" above the first interval on the time line*], is one-seventh of the time, and it would go with … I'm going to put a dotted line up here like this [*draws a dotted line that links the first interval on the time line to a point about a seventh of the way from the left on the distance line and draws a squiggle that connects the left end point to the dotted line, thus creating a interval of sorts*] … [*turns paper so that Ann can see better*] that would go with one-seventh of the distance [*writes "1/7 of the distance" beneath the squiggle*]. Okay?

Excerpt 7 — 00:15:15 to 00:16:31

1. Ann: So we're just … so to find out the answer, you just do what you did before like on the other one [*gestures toward the computer and the scratch paper*]?

2. Pat: I don't know what you did before. Why don't you explain that.

3. Ann: Like this [*points scratch paper with work from opening situation—"Cut up a distance into 5 parts, how would you find the length of each part?"*]. You just divide a hundred by seven.

4. Pat: Yeah!

5. Ann: And you come up with the answer.

6. Pat: So, if it takes him … if one second is one-seventh of the time [*drags pen over the first interval on the time line*], then in that one second he's going to go one-seventh of the distance [*drags pen over first interval on the distance line*]. And if the whole distance is one hundred feet [*points across the distance line to the 100*], then what's this part [*points to the first interval on the distance line*] that he went in one second?

7. Ann: One-seventh … of a hundred.

8. Pat: Try that … I mean how would [*gestures to the calculator*] you calculate one-seventh of one hundred?

9. Ann: [*Looks down at the calculator*] … One hundred divided into seven, or seven divided into a hundred?

10. Pat: Okay. [*Ann uses calculator to calculate 100÷7*] Come up with a strange number?

11. Ann: Yeah.
12. Pat: All right, what is that number?
13. Ann: [*Reading the calculator display*] Fourteen and something something something.
14. Pat: So it's fourteen and how many tenths?
15. Ann: Two.
16. Pat: How many hundredths?
17. Ann: Eight.
18. Pat: All right. How many thousandths?
19. Ann: Five.
20. Pat: If we round it off to thousandths, what would we round it out to?
21. Ann: Six.
22. Pat: [*Writes "14.286" at the top of the page.*]

Excerpt 8 — 00:16:31 to 00:19:01

1. Pat: Okay. So … these [*pointing to "14.286"*] are what? That's fourteen point two eight six what?
2. Ann: Umm, … seconds … Distance? Distance.
3. Pat: Where are we finding it? In time [*points to the time line*] or distance [*points to the distance line*]?
4. Ann: No [*points to the distance line*], in distance.
5. Pat: And we didn't say what this is [*points to "100" on the distance line*]. That's one hundred feet [writes "ft" after 100]. So, what's this segment here [*holds up the sheet so that Ann gets a good view; points to the first interval on the distance line*]?
6. Ann: It's fourteen and … it's fourteen and two hundred and eighty-six thousand feet.
7. Pat: Two hundred eighty-six thousandths of a foot [*writes "ft" after 14.286*; *Pause.*] So if he's going to go a hundred feet [*drags pen across the distance line*] in seven seconds [*drags pen across the time line*], what do we end up saying that he goes each second [points to the first interval on the distance line]?
8. Ann: Hmmm?
9. Pat: Remember. This was [*points to the first tick interval on the time line*] … no. What's your question [*looks at Ann closely*]?
10. Ann: I didn't get what you just said … I didn't, I didn't hear it.

11. Pat: Remember [*moves pen across the distance line*] I started out by saying suppose that it takes him seven seconds to go the whole way [*drags pen across the distance line*]. Then you said that one second [*points to 1 sec over the first tick interval of the time line*] is one-seventh of the time [*points to "1/7 of the time" below the time line*]. So the distance he goes in one second [*drags pen across first tick interval on the distance line*] is one-seventh of the distance [*points to "1/7 of the distance" below the distance line*].
12. Ann: Uh huh.
13. Pat: So how many feet does he go each second [*drags pen across the first tick interval on the time line*]?
14. Ann: He goes [*pause*] fourteen and two hundred and eight-six thousandths of a feet, of a foot each second.
15. Pat: And in the next second?
16. Ann: He would go the same amount of time as in the second before.
17. Pat: Yeah. He's going the same amount of time which is one second.
18. Ann: And the same amount of distance.
19. Pat: Which is?
20. Ann: Fourteen and . . .
21. Pat: You can just say 14 point 2, 8, 6 for now.
22. Ann: 14 point 2, 8, 6.
23. Pat: Okay. And the next second he'll go … [*draws a line from the third tick mark on the time line to a point about halfway along the distance line, making a third tick interval there*]?
24. Ann: 14 point 2, 8, 6.
25. Pat: [Writes "14.286" above the third tick interval on the distance line] And in the fourth second [*draws a line from the fourth tick mark on the time line to the distance line to make a fourth tick mark there*]?
26. Ann: 14 point 2, 8, 6.
27. Pat: Point 2, 8, 6. And tell me what that number is. It's a number of what?
28. Ann: Feet.
29. Pat: Okay. So it's [*writes "ft" after each 14.286*] so each second how far does he go?
30. Ann: 14 point 2, 8, 6.
31. Pat: Okay [*nods*].

32. Ann: So, like, if you divided that number [*points to "100" on the distance line*] by seven you could come up with the answer too, right?

00:19:01 to 00:20:20

1. Pat: Try that. Try giving him a speed of 14.286 feet per second [*points to the Rabbit-speed Box; Ann types "14.286"*]. Before you run the rabbit, what should the timer [*gestures toward the Time Counter*] say when he gets over here [*taps on the 100 ft mark of the on-screen distance line*]?

2. Ann: Umm … It should say [*pause*] … a hundred … er, umm, the timer?

3. Pat: Yes, the timer down here [*points to the Time Counter*].

4. Ann: Seven seconds?

5. Pat: It should say seven seconds, okay, if we're correct. Go ahead and try that [*Ann activates the rabbit*]. Put the arrow over pause [*the pause button on Over and Back*] so that when it gets right near the end you can stop it temporarily. [*Pat and Ann wait for Rabbit to reach the end of the distance line; Ann clicks the Pause button; the display shows that Rabbit went 100.7 feet in 7.052 seconds.*] So he's gone a little bit farther [*points to the 100 ft*] and a little bit more than seven seconds [*points to the Time Counter*]. So it seems pretty close doesn't it?

6. Ann: Yeah.

00:20:20 to 00:21:21

1. Pat: Okay, Mr. B is going to ask you to do some more where … what I want you to do is to think of … okay. He's going to say, "Give it a speed so that it will go over and back [*motions over and back the distance line*] or just over [*gestures across the computer's distance line*] in a certain amount of time." So the way that I want you to think about it [*points to his head*] is that you're cutting up the time [*gestures to the time line*]. And when you do that, you're also cutting up the distance [*gestures to the distance line. Ann nods slightly*]. Okay? And whatever part of the time one second is [*points to the time line*] that's what part of the distance you will go [*gestures across the distance line*]. Like if … if one second is one-twelfth of the time [*gestures across the time line*], how far will he go in distance [*points to the right end of the distance line*]?

2. Ann: [*Pause*] Umm … He would go a twelfth of a hundred feet.

3. Pat: [*Nods*] Exactly! Very good Ann! You've really got it nailed. So Mr. B is going to ask you to practice that kind of reasoning, okay?

4. Ann: Okay.

00:21:21 to 00:22:09

1. Pat: All right. Very good! [*pats Ann on her shoulder*] I'm very proud of you. It's good reasoning … I've got to run to a meeting, okay? [*Pat walks away from the table; he and Bill make plans to talk by phone about today's session. Ann plays with Over & Back, running Rabbit and watching its motion and the timer. Bill takes over*].

00:00:00 to 00:00:45

1. Bill: Ann, you're doing great. [*Ann activates rabbit*] You're seeing how long it will take him to go back, is that it? How long should it take him to come all the way back over here [*points to the computer screen's 0 pt on the distance line*] do you think?
2. Ann: About fourteen seconds. [*Rabbit finishes*]. Close.
3. Bill: Not just about. You're right on the money. I told you that light bulb was pretty close yesterday and that's what I think you've come up with. Umm … [Bill shuffles through some papers. Ann types "35" for the Rabbit-speed Box] Remember the ones that we were working with … here we go. Here's the sheet [*shows Ann her worksheet from Day 2*]. Remember those that we were working with the other day?
4. Ann: Uh huh.
5. Bill: This [*points to Ann's Activity 2 worksheet*] is in effect what you just went through here [*puts hand on the work Pat and Ann just worked on*]. So how about we pick up where we left off there.
6. Ann: Okay.

00:00:45 to 00:04:15

Over and back in 6 seconds. Ann draws a digram (a line segment for distance of 100 feet and a line segment for time), partitions each into 6 segments, and identifies the quantities that each part of her diagram represents. Most discussion is about how long it would take Rabbit to go over and back. The question of what speed Rabbit would need to go over and back in 6 seconds is never answered.

1. Bill: And, based on what Dr. Thompson just covered with you, why don't you see if you can figure out, for example, what the setting should be if you are going to go for a total time of six seconds [*points to Problem 4 of Activity 2, which asks for a speed to make Rabbit go over and back in 6 seconds*].
2. Ann: Okay [*draws lines on a clean sheet of paper*].
3. Bill: Do you want to do it over and back or just over?
4. Ann: Just over.
5. Bill: Okay.
6. Ann: [*Pause. Counts to herself while while making six intervals on distance and time lines; writes "0" and "100" on the ends of the distance line and writes "0" and "6" on the time line.*] Okay, so, we're just talking about six seconds, right? That's it?

PWT 7/20/2004 6:00 AM
Comment [1]: Bill's language is vague. "figure out what setting you should use if you are going to go for a total time of six seconds" instead of "figure out what speed Rabbit should travel at if it is going to go over and back in six seconds."

7. Bill: Uh huh [*yes*]. So what have you got on your graph there?
8. Ann: What I have right here, this is distance [*writes "Dis" at the left end of the distance line*].
9. Bill: Uh huh [*yes*].
10. Ann: [*Pause*] Distance. And, umm … it's split up into six sections.
11. Bill: Okay.
12. Ann: One for each of the each of the distance that he will travel.
13. Bill: Great.
14. Ann: And this is the time [*points to the time line and writes "sec" above it*] and it's also cut up into six sections … because one for each second [*writes 1 through 6 over the appropriate intervals on the time line*].
15. Bill: Good. Right.
16. Ann: [*Pause*] And if that's six [*points to the time line*], this is a hundred [*points to the distance line*]. So it's six into a hundred.
17. Bill: Okay.
18. Ann: [*Writes 100÷6 in long division form*] … So that you could find what the answer is. [*Uses the calculator to calculate 100÷6.*] So it would be … to go over it would probably be 16 point 6 [*writes "16.6" on the right side of the distance line*].
19. Bill: What is that, time-wise?
20. Ann: That would be, umm … six … for distance. Distance. That's only one distance [*circles the 16.6 and draws an arrow from it to the first interval on the distance line*].
21. Bill: Okay … And how long of time is that distance covered in?
22. Ann: One second.
23. Bill: Should we try it and see?
24. Ann: [*Nods*] We're talking about the rabbit, right?
25. Bill: Yeah. [*Ann types "16.6"*] Does it come out to exactly 16.6, because as you can see we can put three decimal places in there if you want to.
26. Ann: Actually it comes out to a lot of sixes.
27. Bill: Oh, okay. Why don't we put two or three of them in there to make it more accurate.
28. Ann: [*Types ".666" after the 16; pulls down a menu to set the display at 3 decimal places; activates Rabbit*]

PWT 7/20/2004 6:00 AM
Comment [2]: Ann said, "One for each of the distance he will travel." She is still, fundamentally, thinking of speed as a length, but has now abstracted from that image the explicit condition that this length is the distance Rabbit will travel in one second, and has represented explicitly the second in which it will travel that distance.

29. Bill: And how long is it going to take him to get here [*points to the 100 ft end point on the computer distance line*]?

30. Ann: It should take him about sixteen point … Wait! To go all the way over?

31. Bill: Uh huh. [*Ann taps pencil repeatedly*] What time did he go over? Look over on your time scale [*points to Ann's scratch paper*].

32. Ann: But that's not the time … It should take him six minutes … six seconds. Six seconds, yeah. But …

33. Bill: Okay. The lower one you had down there [*referring to Ann's time line*], wasn't that the time scale you had?

34. Ann: Yeah, this [*her time line*] is it. This [*points to "6 seconds"*] is how long it would take him. And that's [*points to "16.666"*] how much for each second [*Rabbit has finished running, but Ann has not looked at the screen*].

35. Bill: How long should it take him to go over and back at that speed?

36. Ann: Umm … [*looking at her paper*]. Twelve seconds?

PWT 7/20/2004 6:00 AM
Comment [3]: Ann never determined Rabbit's speed to go over and back. Instead, she determined its time given that it took 6 seconds to go one way.

00:04:15 to 00:04:54

From here to 00:06:35, Bill tries to focus Ann's attention on speed over and back, but his questions are vague, and Ann focuses on calculating Rabbit's distance over and back given that his speed is set at 16.666 feet per second (she calculates 16.6 times 6, and then 99.996+99.996). Bill finally moves on to the next problem without having resolved the issue of Rabbit's speed to go over and back in 6 seconds.

1. Bill: Look what it did [*gestures to the Time Counter*]. You're right on the money. Very good. [*Picks up Activity 2 sheet.*] And you already did the one with Dr. Thompson here [*points to problem 5, "Give Rabbit a speed that makes it go over and back in 7 seconds"*] on the seven seconds. What are we …

2. Ann: So what are we … so what we have to set him for is this [*taps her paper; cannot see at what she points*], right?

3. Bill: Yeah. But on this one [*points to problem 4*], see, this was set up to go over and back, and the ones you were just doing, which is fine, you can do them just like you've done them for just going over. This one [*make Rabbit go over and back in 6 seconds*] you figured out the other day for going over and back was 33.1, but now we're just going over.

4. Ann: That would be easy to figure out. You just times this [points here at "16.6"] by six.

5. Bill: That's right [*nods*]. If you do that, what do you get?

6. Ann: … Umm … [*Uses the calculator, but doesn't say what she is calculating*] Uh huh. Yeah. Wait. Okay, we're trying this [*16.666*] by this [*6*], right?

7. Bill: Yeah, if you multiply the 16.666666 times six, what are you going to get?

00:04:54 to 00:06:35

8. Ann: [*Continues to use the calculator. Writes "99.996" in bottom right hand corner of scratch paper*] 99 point 996.
9. Bill: Yeah, what is that number?
10. Ann: That's … um … the total distance that all of these are altogether [*draws little arcs underneath each interval on the distance line*].
11. Bill: Sure is pretty close to your hundred, isn't it? That's correct.
12. Ann: And you would have to add this [*taps 99.996*] twice to go over and back.
13. Bill: To go over and back, that's correct. [*Ann writes "+99.996" underneath the first 99.996 and adds them*] You can do that on the calculator, you just hit times two.
14. Ann: [*Continues pencil computation; writes"199.992".*] So that's probably how about how long it would take him to go back … go over and back.
15. Bill: You mean that's how much time it would take him?
16. Ann: Err, how much the distance is to go over and back.
17. Bill: [*Nods*] Right. Okay. Very good.

00:06:35 to 00:10:43

Give Rabbit a speed to go over and back in 6.5 seconds. Bill asks the question as "How long will it's going to take him to go over in 6.5 seconds." Ann draws two line segments and partitions each into 6.5 sections, labeling each end interval as "1/2". Bill does not ask about the meanings of the halves; Ann says that each distance segment is one-sixth of 100; Bill mentions that we don't have a name for this, that it would be like "one six point fiveth" of a hundred. Ann tests her prediction; Bill asks about how long it would take Rabbit to go over and back; Ann says "13 seconds." The question of what speed to give Rabbit so that it will go over and back in 6.5 seconds is not addressed.

1. Bill: Let's put that paper aside [*gestures at Ann's scratch paper*] and let's use another one to do this next one. [*Ann gets a clean sheet of paper*] It's a little bit different, but the work will be exactly the same. Try to see how long it's going to take him to go over in six point five or six and a half seconds.
2. Ann: [*Draws a distance line from 0 to 100 and a time line labeled "seconds". Speaks softly.*] 1, 2, 3, 4. [*Pause*] 1, 2, 3, 4, 5, 6 [*put six and a half intervals in each line*]. That's a half right here [*labels the last interval "1/2" on both lines*].

PWT 7/20/2004 6:00 AM
Comment [4]: Bill misspoke. He meant to say, "How fast must Rabbit run to go over and back in 6.5 seconds."

3. Bill: Okay. Fine.
4. Ann: Okay. So we have … [*Long pause; writes something out of camera view.*]
5. Bill: Uh huh.
6. Ann: So this right here would be six five [*writes "6.5" at the right end of the time line*].
7. Bill: Uh huh. Very good. Yes, it's six and a half.
8. Ann: [*Writes "[6 1/2*]" next to 6.5] And each one of … So you'd divide 6.5 into a hundred [*writes 100÷6.5 in long division form above the distance line*].
9. Bill: Sounds like a winner to me.
10. Ann: [*Uses calculator to calculate 100÷6.5*] 15 point 3, 8, 4 [*writes "15.384" on top of the long division*].
11. Bill: Okay. And what does that 15 point 3, 8, 4 symbolize?
12. Ann: That symbolizes this much [*draws a bracket under the first interval on the distance line*] of the whole entire distance [*writes "15.384" under the bracket*].
13. Bill: Okay.
14. Ann: One-sixth of a hundred.
15. Bill: It's not a sixth, is it?
16. Ann: It's uh …
17. Bill: If you divided a hundred by 6 point 5 … so we really don't have a way of saying that unless you want to say it's "one sixth point fiveth" of a hundred. But how far is that now he's traveled?
18. Ann: How far has he traveled?
19. Bill: Yeah, what is that distance there? Fifteen point …
20. Ann: [*Looks closely at her scratch paper.*] Point three eight four.
21. Bill: And how long did it take him to travel that distance?
22. Ann: One second.
23. Bill: [*Nods*] Okay. What is the rabbit's speed going to be that you are going to set [*gestures to the distance line on the computer*]?
24. Ann: To go all the way over?
25. Bill: Yeah.
26. Ann: Umm … this [*points to "15.384"*], right?

PWT 7/20/2004 6:00 AM
Comment [5]: Here would have been a natural place to question Ann! "What about these halves? What does this last segment on the time line represent? What does this last segment on the distance line represent? What does this last distance segment have to do with this last time segment?"

PWT 7/20/2004 6:00 AM
Comment [6]: Ann has the right concept (i.e., a fractional part), but her understanding of fractional part and her understanding of numeration are not well connected.

PWT 7/20/2004 6:00 AM
Comment [7]: Why didn't Bill say "No, to go over AND BACK?"

27. Bill: Let's try it. [*Ann types "15.384" as Rabbit's speed*] Run him … go. [*Ann clicks "Run Rabbit"; Bill and Ann watch the screen.*] Remember to put this thing [*mouse arrow*] over here next to the pause [*points to the Pause button*] so that after you start him running we can stop him right before he gets to the hundred foot mark. [*Ann pauses Rabbit at 99.0 ft and 6.432 sec.*] You got him right at ninety-nine feet and that's pretty darn close, isn't it [*points to the Time Counter*]?

28. Ann: Mmmm …

29. Bill: If you let him go, he should probably … Well, let's ask it this way [*Ann clicks "Resume"; Rabbit continues*]. How long would it take him to go over and back?

30. Ann: About … um …

31. Bill: [*The Time Counter gets up to eleven seconds*] Pause it.

32. Ann: [*Does not pause Rabbit*] … thirteen seconds [*as the Time Counter passes 12 seconds*].

33. Bill: Oop, you got it. That's all right, let it go … [*Rabit finishes*] Pretty darn close, isn't it? The reason this [*points to the Time Counter*] is off by a little bit [*makes a small space with his fingers*] is because we're way out there [*gestures toward the calculator*] in the decimal range. But you've got the picture, don't you?

34. Ann: Yeah.

PWT 7/20/2004 6:00 AM
Comment [8]: Again, Bill asked about "How long would it take him …" instead of "At what speed must he travel …". Is this a result of an image he formed of my instruction, or is it that he is naturally inclined to think about time arising from traveling at some speed?

PWT 7/20/2004 6:00 AM
Comment [9]: The question of what speed to give Rabbit so that it goes over and back in 6.5 seconds still has not been addressed.

00:10:43 to 00:13:00

Give Rabbit a speed so that it goes over and back in 8.3 seconds. Principle topic of discussion is how to interpret ".3". Ann first thinks of it as 1/3 second. Bill's questions help her clarify that it represents 3/10 second. Ann calculates 100 ÷ 8.3 and explains that the answer is Rabbit's speed to go "all the way over" in 8.3 seconds.

1. Bill: Why don't you try one more at 8.3 seconds in time.

2. Ann: 8.3. [*Ann gets new sheet of paper. Draws a distance line from 0 to 100 and a time line from 0 to 8.3, then cuts each into nine intervals. Makes a bracket on the last interval on the time line and writes "1/3"*]

3. Bill: Okay. Careful now, point three is not really the same as one-third [*Ann erases the 1/3*]. Can you read the um …

4. Ann: It's like a quarter kind of right?

5. Bill: Kind of. It's kind of like a quarter, kind of like a part, but it's not either one. Can you read the number 8.3 to me in decimal form. How would I read that as a decimal number?

6. Ann: Eight and three tenths.

7. Bill: Yeah, so it's three-tenths …

8. Ann: Three-tenths.
9. Bill: … you see, not one third. [*Ann writes ".3" under the bracket*] The way you've done it there [*.3*] is fine.
10. Ann: [*Labels the top line "Dis". Makes a bracket on the last interval on the distance line and writes ".3" under it*] Okay, so we need to know how much this is [*draws a bracket under the first interval on the distance line*] and if we divide this number [*points to 100 ft*] by this number [*points to 8.3*] we … get the answer.
11. Bill: Okay.
12. Ann: [*Writes 100÷8.3 in long division form, then uses her calculator.*] And the answer for the amount that we have there would be 12 point 0,4,8 [*writes "12.048" on top of the long division*].
13. Bill: Okay.
14. Ann: That's for just one of these [*writes "12.048" under the first interval on the distance line*].
15. Bill: Right. Good show.
16. Ann: So that's the speed that we should set him at for him to go all the way over.
17. Bill: Okay.
18. Ann: And that would take probably about 8 point 3 seconds.

00:13:00 to 00:13:57

Bill raises the issue of unit of speed. Beth says that 12.048 is a number of feet, and that this number of feet is how far Rabbit will go in one second.

1. Bill: Good. You've got it down cold. Tell me just one more time. What is the speed [*gestures toward the computer*] that you're going to set the rabbit for in this one [*gestures to Ann's scratch paper*]?
2. Ann: 12 point 0,4,8.
3. Bill: What are the units [*makes a space between his fingers*], 12 point 0,4,8 what?
4. Ann: Twelve and …
5. Bill: No, 12 point 0,4,8 [*waves finger at Ann's scratch paper*] …
6. Ann: The units?
7. Bill: Yeah! What are the units that we're using here?
8. Ann: What do you mean like units?

9. Bill: Well, this just says 30 [*points to Turtle-Over Box*], but it's 30 miles per hour or something like that.

10. Ann: Oh, feet. [*Writes "feet in 1 second" next to "12.048" under the first interval; draws box around "12.048 feet in 1 second".*] It's how many feet he will go in one second.

11. Bill: In one second. Okay, how do we say that as a speed.

12. Ann: Yeah, it's a speed.

13. Bill: And how do we say it as a speed? When I'm talking about the speed that your mom drives, it's 55 miles … per … hour.

14. Ann: It's 12 point 0, 4, 8 feet per second.

PWT 7/20/2004 6:00 AM
Comment [10]: Ann says that "12.048" is a number of feet, and later explains that these feet are how far Rabbit will go in one second.

It seems that, fundamentally, speed is still a distance.

PWT 7/20/2004 6:00 AM
Comment [11]: Does she say this to parallel "miles per hour," or is this a consolidation of "feet in one second"?

00:13:57 to 00:15:20

Ann tests her prediction for speed to make Rabbit go over in 8.3 seconds. Bill again asks her about total time, but does not ask her to address the question of at what speed Rabbit must travel to go over AND BACK in 8.3 seconds.

1. Bill: Good show. Do you want to try it [*points to the Rabbit-speed Box*] and see? Then we'll switch over and do something else if your prediction is accurate.

2. Ann: [*Types "12.048" in the Rabbit-speed Box*] Okay. I'll push enter [*chuckles*].

3. Bill: Good. [*Ann presses the "Run Rabbit" button; Rabbit begins moving.*] What's our goal-time on this for 100 feet?

4. Ann: Three point …

5. Bill: 8.3.

6. Ann: 8.3.

7. Bill: Okay. And how long would it take him to go over and back?

8. Ann: [*Pauses Rabbit at 98.8 ft and 8.200 sec.*] It would take him about … um, … oh, I just had it. To go over and back would take him 17 seconds.

9. Bill: Just double that [*gestures at the Time Counter*], right?

10. Ann: Oh, almost.

11. Bill: Yeah.

12. Ann: It would take him, like 16.6.

13. Bill: You're right there. He's 98.8 [*points to Rabbit*] and you're right at it, eight point … You couldn't stop that much closer. Right on the money. [*Ann lets Rabbit resume*] There he goes. [*Rabbit finishes.*] 16 point 6! Boy, you're very accurate, Ann. That's excellent.

00:15:20 to 00:15:49

1. Bill: Let's switch gears and go back to having a race between the turtle and the rabbit [*shuffles through his papers*].
2. Ann: Okay. The turtle can win this race, watch this [*clicks "Run Both"; speeds are out of camera view; Turtle and Rabbit run simultaneously*].
3. Bill: Well, I would hope he can. Yes.
4. Ann: [*Clicks the Stop button. Both animals stop in place.*] They just stay there until you run them again and pick up their other partner and become one again.
5. Bill: I haven't seen that before. You know more than I do. That's good.

00:15:49 to 00:17:49

Bill begins to ask "Turtle goes over at 20 ft/sec and returns at 40 ft/sec. Give Rabbit a speed so that they tie." However, he runs out of time. Anns initial thought is to add Turtle's speeds; she tries that, and Rabbit wins going away.

1. Bill: Okay, take a look at this and let's see if we can make sense out of this [*shows Ann Activity 3*]. Here [*points to the first line on Activity 3, where Rabbit's speed is missing*], we're going to set the turtle at 20 over and 40 coming back. And the distance over is 100 feet. Okay? In fact, no, let's go inside [*points to the computer screen*]. Open the options menu and change the distance to 100 feet total [*Ann proceeds use the mouse to get the race to go only 100, rather than 200 ft*]. See where it says "set distance"? Whoops. You have to come down again, I guess. No, we can't do it. There we go.
2. Ann: [*Ann does not change the distance*] It's just 100 feet only?
3. Bill: Yeah, that's what they want it. That's okay then I guess. We'll have to remember that he's running over and back now, that's the time that we're talking about so really the total distance is how long?
4. Ann: Two hundred feet.
5. Bill: [*Nods*] Okay. So if we set the turtle for 20 …
6. Ann: Uh huh.
7. Bill: … going over and 40 coming back.
8. Ann: Uh huh.
9. Bill: [*The bell rings*] … Oh, we just ran out of time. We could pick up on this tomorrow, but here's what we were leaning to the other day. If we had the turtle going over at 20 and

back at 40 and the distance over is 100 and back is 100, what speed do we have to set the rabbit at?

10. Ann: So, he can win?
11. Bill: So they'll tie.
12. Ann: So they'll tie!
13. Bill: Yeah. Okay.
14. Ann: Umm …
15. Bill: We'll pick up on that tomorrow, okay.
16. Ann: Okay.
17. Bill: Because we are going to be out of time and I know you've got to get to your next class and so do I. But, you did excellently today, my dear. That's very good.
18. Ann: [*Sets Rabbit's speed to 60, Turtle-Over Box to 20, and Turtle-Back Box to 40*] I just want to see something [*activates race*].
19. Bill: Who's going to win that one?
20. Ann: [*Pause. Rabbit wins*] Well, so adding them together won't work. [*Changes Rabbit's speed to 15*] Okay.
21. Bill: Okay. Well thank you very much.
22. Ann: Okay.
23. Bill: We'll get those papers and then we'll be out of here.