Bill and Alba met the previous afternoon (see Tchr. Mtg. #29, 5/1/91). In that meeting Bill described his understanding of what Pat had done, but he and Alba did not talk about what he would do in this session.

00:00:00 to 00:01:37

Bill asks Ann to describe what they had done yesterday. Ann is not very coherent: she says that they were finding the time from a distance and a time.

- 1. Bill: Well, where did we leave off yesterday, can you tell me?
- 2. Ann: Umm, we were doing ... that [points to the paper in front of Bill. Bill laughs]. We started that.
- 3. Bill: What's that [pretends to hide the sheet]?
- 4. Ann: Um, it's ... [tries to look at the paper]
- 5. Bill: I'm not trying to hide it from you [*Ann chuckles*]. Before we get to this [*puts hand so that it covers the sheet*], what did we do yesterday, do you remember?
- 6. Ann: We were talking about time and how--and I figured out how you get time ...
- 7. Bill: Uh huh.
- 8. Ann: ... by using just a hundred feet [*points to the 100 ft mark on the computer screen*] and the seconds.
- 9. Bill: Okay [*nods*]. And you were making some comparison [*holds hands apart*] of the time and the distance too, right?
- 10. Ann: Yeah.
- 11. Bill: Okay. The last problem we did yesterday, I think, maybe, well, maybe it wasn't. Well, let's just do one review problem, okay? Let's say that, remember that sheet [*points to his stack of papers*] we were working on yesterday we said with the rabbits, I mean if the rabbit, or I was going to say the hare. If the hare goes over [*moves finger in the air over*] and back in some number of seconds how fast is he going? Do you think you could still do one of those?
- 12. Ann: Yeah.
- 13. Bill: Okay. Let's say he goes over and back in ... 11.5 seconds.
- 14. Ann: Over and back?
- 15. Bill: [Nods] Uh huh.

- 16. Ann: Okay.
- 17. Bill: Okay.
- 18. Ann: In 11 seconds?
- 19. Bill: How fast does he have to go to do that?
- 20. Ann: Okay, so ...
- 21. Bill: And we've got paper there [*points to Ann's pile of scratch paper*] and a calculator here [*hands Ann the calculator*]. I brought you the scratched one [*calculator*] again. Sorry about that.
- 22. Ann: Okay, a hundred, a hundred divided into 11, right? [Calculates 100÷11].
- 23. Bill: I said yes, 11.5.
- 24. Ann: Oh, making it tough [calculates 100÷11.5].
- 25. Bill: Now we're talking about going over and back now, remember?

00:01:37 to 00:04:30

Give Rabbit a speed to make it go over and back in 11.5 seconds. Ann calculates a speed to go over in 11.5 seconds, then doubles the speed. Bill asks her how long it should take Rabbit to go just over; Ann says, "half of 11.5 seconds."

- 1. Ann: Okay, let's say it was just, since we only have this much [*points to the Time Counter*], 8.6, okay? And ... uh ... then ... you do 8.6, then you do 8.6, 8.6 plus, plus 8.6 equals [*uses the calculator to calculate 8.6+8.6*]. So it would come out to about 17.2 seconds. For over and back.
- 2. Bill: Okay, we have the seconds now, remember?
- 3. Ann: I mean feet, feet. Wait a second. What are we doing? [Chuckles].
- 4. Bill: Let's go back. Get a piece of paper there [*points to the pile of scratch paper*]. Why don't you make some notes like you did yesterday so you don't forget where we are.
- 5. Ann: I know ... [Shakes head] No, I know where ... Okay, so what we're doing ...
- 6. Bill: Okay, he's going to go over and back [*gestures over and back*] in a total time of eleven and a half seconds. 11.5 seconds.

- 7. Ann: So he'll have to go 8.6, um, feet [moves her hand across the distance line] to get here [points to the right end of the distance line] in eleven point ... eleven seconds. Right?
- 8. Bill: I'm not sure.
- 9. Ann: I don't
- 10. Bill: I don't know how you calculated this [*points at the computer screen*]. If you tell me what you did maybe ... [*Ann reaches for the mouse*] We can try and find out, that's for sure.
- 11. Ann: Wait, 8.6 ... [enters "8.6" into the Rabbit-speed Box]
- 12. Bill: Okay. [*Ann activates the rabbit*] So you're saying it's going to take him eleven and a half seconds to get to 100 feet, in ...
- 13. Ann: Yeah.
- 14. Bill: At that speed.
- 15. Ann: So ... so if you put it at 17, wait ... 8.6 [*Ann again adds* 8.6+8.6 on the *calculator*].
- 16. Bill: [Watching Rabbit run as Ann calculutes.] You were right, it turned at 11.5.
- 17. Ann: 8.6 plus 8.6 equals ... so you'd get ... so you he'd have to go ... to be 11 seconds it'd have to be 17.2.
- 18. Bill: At his speed, you mean?
- 19. Ann: Yeah.
- 20. Bill: Okay.
- 21. Ann: At his speed.
- 22. Bill: Very good, you got it. Okay, let's go on from there then. [*Ann types "17.2" into the Rabbit-speed Box and activates the rabbit.*] You want to try that one, that's fine. [*Both watch Rabbit run; Bill points at right end of Distance line while Rabbit is 1/3 way from it.*] What should his time be when he turns the corner there?
- 23. Ann: Half of eleven point ... half of 11.5, yeah.
- 24. Bill: Good. [Ann pauses Rabbit at 169.6 ft] You stopping?
- 25. Ann: I just want to see how many feet he was going [clicks "Resume"].

26. Bill: Oh. Okay. Very good. Pretty darn close. If we'd put in some more decimal numbers [*for Rabbit-speed*] it would have been right on the money.

00:04:30 to 00:06:06

Turtle goes over at 20 ft/sec, back at 40 ft/sec. Give Rabbit a speed so that they tie.

Ann attempts to embelish her image of the situation by imagining what would happen if she set Rabbit's speed at 40 ft/sec. She says that this wouldn't work, since Rabbit would get to the end first and then Turtle would come back behind Rabbit with no way to catch up. Bill, however, wants her to stop what she is doinng and "think about it without trying to guess or estimate."

- Bill: Okay, let's go on from there now. This is what we started to go on yesterday when that PE bell rang [shows Ann Activity 3]. If we had the turtle set at 20 ... [points to the first line of Activity 3, under the column labeled "Turtle >"] ... want to change him to 20? [Ann types "20" for the Turtle-Over speed] That's his over speed [points at "40" in the column labeled "Turtle <-". Ann types "40" for the Turtle-Back speed] Back. No, wait a minute. That's right, 40. Now I need you to tell me ... the distance he's going to run is [points to the "Distance" column on Activity 3] 100 feet.
- 2. Ann: Uh huh.
- 3. Bill: Okay? How long, or at what speed, I mean, do I have to set the rabbit for?
- 4. Ann: So they'll both end ...
- 5. J&B: ... at the same time.
- 6. Bill: Uh huh.
- 7. Ann: Well, it's hard to say [*Both chuckle. Ann pauses, looking at the computer screen*]. Okay, you've got a hundred and 20 one way for the turtle, right? A hundred and 20 this way ... [*moves mouse pointer over the distance line*]?
- 8. Bill: [Looks closely at the screen.] You lost me there, would you repeat that.
- 9. Ann: He has to go 20 this way, right [moves mouse pointer across the distance line]?
- 10. Bill: Yeah, he's going 20 feet over ... 20 feet per second over.
- 11. Ann: And then 40 back [drags mouse pointer back along the distance line].

- 12. Bill: Right. [*Ann activates the turtle*] So instead of trying to figure out the problem yet ...
- 13. Ann: So he'd only go that ...
- 14. Bill: Now stop it for ... [points to the on-screen buttons]
- 15. Ann: [*Ignoring Bill*] I can't set the rabbit at 40 because [*drags finger over the distance line*] then he would go over here [*points to the 100 ft mark*] and he'd [*Rabbit*] reach it before he [*Turtle*] would, so that he [*Rabbit*] would go back [*gestures back along the distance line*] faster.
- 16. Bill: Okay, part of the problem we have now, you've got to think about the problem, instead of trying to, uh ...
- 17. Ann: ... solve it?
- 18. Bill: ... guess or estimate. How are we going to approach the problem?
- 19. Ann: Umm ...
- 20. Bill: Don't forget what you learned yesterday and what you were doing yesterday, because that's in effect, what you need to know to solve this problem.
- 21. Ann: ... Okay [moves the mouse pointer around on the screen].

00:06:06 to 00:08:17

Bill asks Ann what she needs to know to set Rabbit's speed. Ann says distance and time, and that they have both. Bill is surprised that she thinks they have the time. Ann points at the Over and Back timer, which reads "7.5 seconds" from Ann's having run Turtle with the given speeds. Ann calculates 100÷7.5, getting 13.3, then doubles that to get 26.6 for Rabbit's speed.

- 1. Bill: Let me ask you this. What do you need to know to be able to set the speed for the rabbit?
- 2. Ann: The time and the distance.
- 3. Bill: We've got the distance, don't we [*points to the Distance column on Activity* 3]?
- 4. Ann: [Looking at the computer whose Time Counter reads 7.5] And we've got the time too.
- 5. Bill: We do?

- 6. Ann: Yeah, right there. [*Looks down at the calculator*] It takes the turtle 7.5 seconds. So it has to take him [*Rabbit*] 7.5 seconds, right?
- 7. Bill: Okay [nods]. Very good.
- 8. Ann: So, it's just almost the same thing that we did yesterday, right?
- 9. Bill: Almost, yeah. But remember we're taking about [points to the computer] over and back now, not just over.
- 10. Ann: Over and back [*clears the calculator display*]. But that [*points to the Rabbit-speed Box*] takes him over and back [*gestures over and back along the distance line*]. That [*Time Counter*] was over and back [*Bill nods*], the 7.5 seconds. So ... [*Calculates 100÷7.5.*] Uhh ... 13.3. You would have to set the rabbit at, so he would go over and back, err, go over at that same time.
- 11. Bill: Okay, but we need him to go over and back.
- 12. Ann: [*with Bill*] and back! So we have to add 13.3. [*Uses the calculator to calculate 13.3*2.*] 13.3 times 2 equals ...
- 13. Bill: Okay, if you multiply 13.3 times 2 ...
- 14. Ann: So it would have to be 26.6.
- 15. Bill: Okay, now that's going to make him tie, right?
- 16. Ann: Yeah [nods].
- 17. Bill: Okay, let's give it a try. [*Ann types "26.6" in the Rabbit-speed Box*] Do you have a whole bunch of sixes in there? [*Looks at the calculator display*] No, just 26.6. Okay.
- 18. Ann: [*Clicks "Run Both"; clicks "Pause" when Rabbit is at 99.9 ft and Turtle is at 75.8 ft.*] See ... [*watching the turtle gain on the rabbit after it turns around*]. He'll catch up!
- 19. Bill: [*They tie*] Very good. You were correct in there [*points to the Time Counter*, 7.5 sec.], that the time was the controlling factor.

00:08:17 to 00:10:31

Bill raises the matter of determining Rabbit's speed without first running Turtle to get his total time. What catches Ann's attention, though, is her observation that Rabbit's total distance is 200 feet; it occurs to her that she could have calculated 200÷7.5, and does so on her calculator to compare it with her original answer. Bill digresses to talk about "bar notation" to show that 26.666666666... can be

written as $26.\overline{6}$. The matter of determining Turtle's total time without running it, though, is dropped. Bill goes on to the next problem.

- 1. Bill: Is there a more efficient way of figuring out the rabbit's speed [*points to the Rabbit-speed Box*] then what you did?
- 2. Ann: Then having to run the turtle [waves hand over and back] and find out.
- 3. Bill: Well, you can't, let's say you can't do that next time. I won't let you [*shakes head*] do that next time.
- 4. Ann: [Whining] No, that's not fair! [Chuckles].
- 5. Bill: [Laughing] Sure it is.
- 6. Ann: No it's not.
- 7. Bill: But besides that,
- 8. Ann: 'Cause I've only ...
- 9. Bill: ... once, once you know the time, over and back,
- 10. Ann: Oh ... !
- 11. Bill: Is there a more efficient way of determining the rabbit's speed [gestures to the computer] for the over and back?
- 12. Ann: Once you know the time?
- 13. Bill: [Pause.] Yeah, you know his time.
- 14. Ann: Okay, so say we know the time and we know the turtle's speeds and we know the distance [*counts the different pieces of information on her fingers*], right?
- 15. Bill: Uh huh [nods].
- 16. Ann: No, ...
- 17. Bill: Well, you know the turtle's speed [*gestures to the computer*] and from that you can get his distance. I mean, I'm sorry, his time. If you know the turtle's speed over [*points to the Turtle-Over Box*] and you know the turtle's speed back [*points to the Turtle-Back Box*] then you can determine the turtle's time [*points to the Time Counter*], right?
- 18. Ann: Yeah.
- 19. Bill: Now, we know the time and we know the distance for the rabbit [*moves hand over and back*]. What is the distance for the rabbit [*looks intently at Ann*]?

- 20. Ann: Uhh, 200 feet if you're going over and back.
- 21. Bill: Right [*nods*]. Now we know the distance [*gestures to the computer*] and we know the time [*again gestures to the computer*] ...
- 22. Ann: So, it would have just been simpler [*points to the computer screen*] just to put in 200 feet divided by 7.5 [*taps on the desk*]. Instead of doing 100 ...
- 23. Bill: Let's try it and see what that does. Very good.
- 24. Ann: [*Uses the calculator to calculate 200*÷7.5.] That would be twenty-six six. That's the same thing.
- 25. Bill: Yeah, 2.666666. Sure because before it was like 1.333, right?
- 26. Ann: Yeah.
- 27. Bill: or 13.3, pardon me. Good show! Okay, so we got that one [*points to the empty first line Rabbit box on Activity 3*], right? What was his speed? Do you have a pencil with you? If you don't, I'll let you use a pen
- 28. Ann: No
- 29. Bill: There you go [hands Ann Activity 3 and a pen].
- 30. Ann: So, this would be the 26 [writes "26 in the empty space]?
- 31. Bill: Uh huh. And you could put a .6 [*Ann puts ".6" after the 26*] and before you put any other numbers down, do you know the notation called bar?
- 32. Ann: Bar?
- 33. Bill: When you put a bar above that second six.
- 34. Ann: No.
- 35. Bill: Okay. It's a notation that we use whenever [*points to the calculator*] you have a number that continues like this [*note: the number on the calculator display*]. Instead of writing a whole bunch of them we just put a bar [*draws an imaginary bar in the air*] above the six, the second six over there [gestures to Activity 3. Ann then draws a line above the decimal place six].
- 36. Ann: So you put the sixes ...
- 37. Bill: So put a line above it, a line above that one's fine. Just put a line above it. And what that tells the reader is that this number is continuous, it just keeps right on going.
- 38. Ann: Oh.

- 39. Bill: Okay?
- 40. Ann: Okay.

00:10:31 to 00:13:33

Turtle goes over at some speed and comes back at 70 ft/sec. Rabbit travels both ways at 30 ft/sec. Give Turtle a speed over so that it and Rabbit tie.

Ann is stuck at first. Bill says, "Just think about the information we have." Ann sees quickly that she needs Rabbit's time, and that she can then find the time Turtle must use to go just over. In the process, she wonders by what to divde Rabbit's distance. She calculates $200\div30$, getting 6.6 . Bill asks her what each of 200, 30, and 6.6 stand for. As she explains, she stops, saying "Wait a minute. This is science. Are you trying to trick me here?" [¶s 30-34]. (Comment on this in the article.) Ann explains that 6.6 is the time it will take Rabbit to go over and back.

- 1. Bill: Well, okay, let's go on to the second. What do we have there [gestures again to *Activity 3*]? Hm, that's a little bit different, isn't it? ["The turtle is going over at some speed and coming back at 70 ft/sec. Rabbit travels both ways at 30 ft/sec. Give Turtle a speed over so that it and Rabbit tie."].
- 2. Ann: Whoa, how can they tie if we have this! [Chuckles.]
- 3. Bill: Well, let's think about it for a second. Let's go through what we know and put in the information that we know ...
- 4. Ann: Oh, I get it! It's the exact same thing, except, except you have the rabbit and you have to figure out the turtle, so all you do is the same thing that we've done with this [*points to the problem they just did on Activity 3*], but instead of doing it with this and this [*points to Turtle's over and back speeds in previous problem*] we do it with this and this [*points to Rabbit's speed and Turtle Back speed in current problem*].
- 5. Bill: Okay [*nods*]. And again this is over. [*Ann laughs*] Yeah that's fine. This is over and back again, so ...
- 6. Ann: So, it would be 200.
- 7. Bill: Right.
- 8. Ann: [Uses the calculator, pressing "200÷"] Two hundred divided by ... Wait a second. I haven't figured out the time. So, I need the time too, right?
- 9. Bill: Uh huh [nods].

- 10. Ann: So that would be ... [gets a piece of scratch paper] time. Okay I have a hundred [writes "100"].
- 11. Bill: Okay, which one are you going to calculate first now?
- 12. Ann: [Writes "30" in front of the 100] The time [looks at Bill].
- 13. Bill: Yeah, for which, the rabbit or the turtle. What?
- 14. Ann: The rabbit.
- 15. Bill: Okay [nods]. And how far is he going to run?
- 16. Ann: He's going to run ... over and back.
- 17. Bill: And how far is that?
- 18. Ann: [Changes the 100 to 200] 200.
- 19. Bill: Okay [nods].
- 20. Ann: And, umm, then 30 ... But would you divide it?
- 21. Bill: Gee, that was the very first thing we were doing the other day. Think about it for a minute. Remember when you first started out here [*gestures to Ann's paper*], how were you determining how long it was going to take them?
- 22. Ann: [Draws a long division symbol around the 200] This way. [taps the 30 and the 200, then chuckles].
- 23. Bill: And you can use a calculator for that, too.
- 24. Ann: Umm. There. [Calculates 200÷30] 200 divided by 30 equals ... So that would be [*reading the display*] 6 ... [*writes "6.6" with a bar over the decimal 6*].
- 25. Bill: Okay. And this is what now? What did you figure out? What is the 6.6 for?
- 26. Ann: Okay, this is feet [*writes "feet" next to 200*], this is distance [*writes "distance" just below the 30*], and this, seconds [*writes "seconds" next to 6.6*].
- 27. Bill: Okay, the 30 is what? Distance?
- 28. Ann: Distance, err, like the speed. A speed.
- 29. Bill: Oh, okay [Ann scratches out distance and writes "speed" beneath the 30]. Okay, and the 6.6 seconds is what?
- 30. Ann: Wait a second ...
- 31. Bill: What does that tell us?

- 32. Ann: [Pause.] This is science.
- 33. Bill: Hm?
- 34. Ann: This is science. Are you trying to trick me here?
- 35. Bill: [*Shakes head*] No, no [*chuckles*], it's also math. Did you know that math is the language of science? [*Ann crosses out the word "feet" next to the 200 and replaces it with "distance"*]. Yeah, this is really physics, but we didn't want to tell you.
- 36. Ann: Yeah.
- 37. Bill: It's like that, yeah.
- 38. Ann: [*Something unintelligible*] ... Yeah. The distances comes up like over time or something like that.
- 39. Bill: That's right.
- 40. Ann: That was like in Mr. Wisser's class or something.
- 41. Bill: Uh huh.
- 42. Ann: Okay. And this [*note: the 6.6*] tells us how long it will take him to go ... over ...
- 43. Bill: Excellent ... and back
- 44. Ann: ... and back.
- 45. Bill: Good!
- 46. Ann: Like this [*circles the 6.6 seconds and draws "<-->", representing over and back and circles it*].

00:13:33 to 00:21:05

Ann first calculates $200\div6.6$, getting 30.3 and says that this is Turtle's overspeed. Bill talks about the calculations she has done, but does not connect them with the quantities Ann's calculations were intended to quantify or the context which led to them.

Bill points out that Ann calculated $200\div30$ to get Rabbit's time. Ann says, "Oh, it should be $200\div70$." Bill asks her what that calculation would give her [¶ 25] and why she would divide 200 by 70 [¶ 27]. Ann says that "That number [70] wasn't

just put there to sit. It has to be doing something!" [¶ 28]. Ann "realizes" that she should calculate $100\div70$, getting 1.4. Ann says that it is 1.4 ft/sec. Bill points out that, before, she divided distance by speed to get time; Ann restates her answer as 1.4 seconds. Bill asks what takes 1.4 seconds; Ann says it takes Turtle 1.4 seconds to go back. She then calculates the time Turtle needs to "fill up" by going over (getting 5.2 seconds). Bill turns again to calculations and vague questions and inaccurate statements [¶s 75 ff, esp. ¶ 91]. Ann calculates $100\div5.2$, getting 19.2 and enters that number as the Turtle-over speed.

- 1. Bill: Now, what are the other times that we have up there? First of all we've got the rabbit here set for 30, right [*types "30" into the Rabbit-speed Box*]?
- 2. Ann: Yeah.
- 3. Bill: And we don't know this one [*Turtle-Over speed*] yet, so we'll just leave that one blank [*erases Turtle-Back speed*], and ...
- 4. Ann: That's 70 [Turtle-Back speed].
- 5. Bill: The back way is going to be 70 [*types "70" for the Turtle-Back speed*]. What speed ... are you going to have to set the turtle to go over at ... to be able to tie [*selects Turtle-Over box to highlight its lack of an entry*]?
- 6. Ann: Okay, now we have 6.6. I'm just gonna, oop ... to do this real simple here. [Uses the calculator to calculate 200÷6.6] 200 divided by 6.6 ... 30.3.
- 7. Bill: For which, now?
- 8. Ann: For, this right there [points to the Turtle-Over Box on the computer].
- 9. Bill: Now, remember he's coming back [*points to the Turtle-Back Box*] already. I mean that's set, we can't change that one.
- 10. Ann: Yeah, but to tie he has to do 30.3. Thirty-thirty actually.
- 11. Bill: Isn't that the rabbit's speed now [points to the Rabbit-speed Box]?
- 12. Ann: No.
- 13. Bill: I mean that's what you came up with, I mean that's how you got the rabbit's speed here [*points to Ann's scratch paper*] originally, right? I mean time, pardon me.
- 14. Ann: But, see ... I trust the calculator [*laughs*].
- 15. Bill: Well, I don't doubt that, but see [pointing to Ann's scratch paper] what you've done [points at 200÷6.6=30.30] is just the reverse of this [points at 200÷30=6.6, the first calculation she did on her scratch paper]. You divided 200 by 6.6 and you got 30.

- 16. Ann: So, it's like the same thing? But it's not right, is it?
- 17. Bill: Uh uh [shakes head "no"].
- 18. Ann: No. So! So I know ... [taps desk several times].
- 19. Bill: Okay. I'll tell you what. Let's use a piece of paper [*points to Ann's pile of scratch paper*] or the bottom part of that [*gestures to Ann's current scratch paper*] would be fine. Let's have you kind of diagram out, like you were doing the other day?
- 20. Ann: Oh, well, no [chuckles].
- 21. Bill: Part of ... Well part of it, because we've got ... we've got to figure out some way to have this combination [points back and forth between the Turtle-Over and Turtle-Back Boxes] of times taken into effect. And what you just did [gestures to Ann's paper] would ... say, okay if the turtle was going over and back [waves hand over and back] ...
- 22. Ann: Ohhhhh!
- 23. Bill: ... at 30 and 30 [*points to the Turtle-Over and Turtle-Back Boxes*], fine they would tie.
- 24. Ann: Ohhh, I get it. So it would be 200 divided by 70.
- 25. Bill: What will that give you?
- 26. Ann: Umm ... I don't know. [Calculates 200÷70] 200 divided by 70. It's 2.8.
- 27. Bill: Okay, ask ... Tell me this: Why did you divide 200 by 70?
- 28. Ann: Because ... [*staring at the computer*] that number wasn't just put there to sit. It has to be doing something.
- 29. Bill: Well, yeah, but why did you use ... I understand why you're dividing by 70 [*points to the Turtle-Back Box*], because that's what you did with the rabbit.
- 30. Ann: Why did I use 200?
- 31. Bill: Yeah.
- 32. Ann: Because that's how my, how, over ... that's wrong. It needs to be a hundred.
- 33. Bill: Aha.
- 34. Ann: I get it.
- 35. Bill: Okay.
- 36. Ann: It has to be [calculates $100 \div 70$] 100 divided by 70.

. . .

- 37. Bill: Okay.
- 38. Ann: 1.4.
- 39. Bill: 1.4 something or other. What is that [gestures to the calculator] now, that's
- 40. Ann: That's 1.4 feet per second.
- 41. Bill: And here [reaches over and points at Ann's scratch paper problem, "200 distance÷30 speed=6.6 seconds"] you divided distance ...
- 42. Ann: Yeah.
- 43. Bill: ... by speed and you got time, ...
- 44. Ann: Uh huh.
- 45. Bill: ... seconds.
- 46. Ann: And that was ...
- 47. Bill: This one you divided distance ...
- 48. J&B: ... by speed.
- 49. Bill: This [points to the calculator display] is what?
- 50. Ann: Time.
- 51. Bill: And what time is that?
- 52. Ann: It's ... 1.4.
- 53. Bill: And 1.4 is ... the time that [gestures to the computer screen] ...
- 54. Ann: Seconds.
- 55. Bill: Yeah.
- 56. Ann: That it takes him [*Turtle*] ...
- 57. Bill: To do, what?
- 58. Ann: To go over and back. To go back, I mean.
- 59. Bill: Okay.
- 60. Ann: So, if it takes him 1.4 seconds to go ... back ... [writes "1.4 <- " on her scratch paper] ... then ... 1.4 out of 6.6 [writes "6.6-1.4" in column form]. Right?

- 61. Bill: Super.
- 62. Ann: And then that will give you the time ... the time that he has to fill. And then you times that by a hundred.
- 63. Bill: Oops. Go one step at a time.
- 64. Ann: Or divide by ... a hundred or something like that.
- 65. Bill: [Nods] Okay.
- 66. Ann: [*Calculates 6.6-1.4=5.2 on paper*] So ... 5.2. [*Circles 5.2*] Those are the seconds that you need to ... umm ... fill up.
- 67. Bill: Uh huh [nods].
- 68. Ann: That's how much you need ... to get ... over.
- 69. Bill: Very good.
- 70. Ann: So
- 71. Bill: So, how do we figure out his speed [gestures to the computer] that will give him that time [gestures to Ann's paper]?
- 72. Ann: [Looks at Bill] Divide it by a hundred?
- 73. Bill: Is that what you did the other day?
- 74. Ann: Yeah. No. Maybe. I don't know.
- 75. Bill: Remember those two diagrams you were doing yesterday? We had the distance and we had the time lengths [*gestures as if making two lines on the paper*].
- 76. Ann: Yeah.
- 77. Bill: We have something like 5.2 ... what did we have [*leans over to see Ann's paper*] ... 5.2 seconds [*pretends to write 5.2 seconds beneath the imaginary bottom line*]?
- 78. Ann: Yeah.
- 79. Bill: What did we do with the, uh, distance line?
- 80. Ann: We marked it off the same [pretends to cut up the line into intervals].
- 81. Bill: Aha.
- 82. Ann: ... So

- 83. Bill: So, what did you essentially do to that line?
- 84. Ann: Huh?
- 85. Bill: Did you multiply it, did you divide it by 100? What, what were you doing to the line [*pretends to cut the line up with hand*] when you showed those ...?
- 86. Ann: I don't know.
- 87. Bill: Think about it for a minute.
- 88. Ann: [Pause.] I was dividing!
- 89. Bill: Yeah. That makes sense to me. Does it make sense to you?
- 90. Ann: Yeah.
- 91. Bill: Okay, now what would you divide that line into, or, I'm sorry, what would you divide the line by?
- 92. Ann: Time?
- 93. Bill: Yes [nods]. That was the other one [again pretends to have lines on his paper]. And what time is that [gestures to Ann] representing?
- 94. Ann: 5.2?
- 95. Bill: [*Nods*] Let's do it and see what happens [*Ann calculates 100÷5.2*]. What did you get?
- 96. Ann: That's 19.2
- 97. Bill: And what would that be?
- 98. Ann: Umm ... It would be ... it would be the speed [writes "19.2 sec" then crosses out the "c" and replaces it with a "p" and then circles all of it].
- 99. Bill: Okay. [Ann writes 5.2÷100 in long division form]. Take a look at your division that you just showed me. Does that make sense?
- 100. Ann: No [chuckles].
- 101. Bill: Okay.
- 102. Ann: Not really.
- 103. Bill: Because that isn't what you did on the calculator. [Ann rewrites the division 100÷5.2] Okay, good. 5.2 into 100 gives you 19.2. Good! You want to set him and let's see if you're right.
- 104. Ann: Okay [types "19.2" into the Turtle-Over Box]. It's gonna be racing back.

- 105. Bill: The turtle's going to really speed back isn't he?
- 106. Ann: Yeah, [activates race] but he's going to go slow in the beginning.
- 107. Bill: Uhh huh [as he and Ann watch the race].
- 108. Ann: But he's going to run himself back.
- 109. Bill: Why don't you get ready to pause him right before they get to the end here [points to the 0 ft mark] and we'll see ... [Turtle gains rapidly on Rabbit] Shewww!
- 110. Ann: [Pauses the race when both animals are near the end.]
- 111. Bill: Looks to me like you got it; look at these distances [points to displayed distances for Turtle and Rabbit; Ann reactivates the race for the final tenth of a second]. How about that? You want to fill it in [gestures toward Ann's activity sheet]. You just won the prize.
- 112. Ann: Yeah. So it was 19.2 [*writes "19.2" in the answer box on her activity sheet*]. Okay.
- 113. Bill: That wasn't too hard was it?
- 114. Ann: No.

00:21:05 to 00:27:12

Bill's questions seem to be oriented toward having Ann use the correct numbers in correct calculations. Whatever he asks, if Ann responds by naming the number Bill has in mind, or by naming the quantity Bill has in mind, or by stating the operation Bill has in mind, then Bill says "Sounds good to me" and ceases his questioning.

Ann continues using the metaphor of finding the amount of time Turtle needs to "fill up." She determines the amount of time so that Turtle will go at the proper speed to use that amount of time in coming back. She calculates $200\div40$, $100\div52$, and 5-1.9 to determine Rabbit's total time, Turtle's time over, and Turtle's time back. She then calculates $100\div3.1$ to get Turtle's back-speed so that it and Rabbit tie.

1. Bill: Good. I told you when the light bulb went off these were going to be really easy. You want to try the next one down there?

- 2. Ann: [*Gets out a new piece of scratch paper*] Fifty-two going this way [types "52" in Turtle-Over box]. And that way is blank [*erases the number in the Turtle-Back Box*].
- 3. Bill: There you go.
- 4. Ann: And that is 40 [*types "40" in the Rabbit-speed Box*]. Okay and then there's a hundred [*the distance*]. So now we need to find a way for the turtle going back instead of going ...
- 5. Bill: [*Nods*] Uh huh.
- 6. Ann: ... forward. Okay. We just do the same thing, right?
- 7. Bill: Pretty much. Yeah. We're going to add one little twist to it here in a minute, but don't worry about that. We're just going to change the distance, but I don't think that's going to bother you.
- 8. Ann: Okay. So ... we'll ... divide 100 by 40? Or 40 by 100, or something like that?
- 9. Bill: Okay, think about what you're telling me then explain to me why you would do that.
- 10. Ann: You'd do that because ... when you divide speed by distance you get time. And we need to know the time so that we can figure out how much time we have to fill ... when he's coming back ... to get the right time as the rabbit.
- 11. Bill: All right. Uh, when you divide, you said divide speed by distance [Ann writes 100÷40 in long division form] or distance by speed. [Looks down at Ann's scratch paper] What you did there was divide distance by speed, okay?
- 12. Ann: Yeah [nods head vigorously].
- 13. Bill: I'll buy that. Why [*gestures to Ann's paper*] did you use 100 in this case for the distance?
- 14. Ann: Because we're only talking about back, not forward and back.
- 15. Bill: Fo-for which one then?
- 16. Ann: [Pause. Looks confused] What?
- 17. Bill: [Looks at the computer screen] For the rabbit or the turtle?
- 18. Ann: Turtle! Rabbit. [*Looks down at her scratch paper and Activity 3*] I don't know.
- 19. Bill: Okay, we're talking about the rabbit, because that's where the 40 came from. How far is he going to run?

- 20. Ann: Over and back?
- 21. Bill: Uh huh.
- 22. Ann: So you need 200?
- 23. Bill: Sure do [nods]. Very good. [Ann tries to change the 100 to 200, but then crosses the whole problem out. She rewrites it as 200÷40 in long division form] Okay.
- 24. Ann: [Calculates 200÷40] Five? [Looks at Bill].
- 25. Bill: Uh huh [*nods*]. [Ann writes "5" and circles it as the answer to 200÷40] Okay. And that's 5 ... seconds, right?
- 26. Ann: Uh huh [*writes "sec." after the 5, "speed" below the 40, and "dis." below the 200*]. Okay. So we know that all together [*puts her two hands together*] it's going to take ... 5 seconds.
- 27. Bill: [Nods] Okay.
- 28. Ann: Which isn't a lot of time. And, um [*pause. Looks down at her scratch paper*] and 5 seconds ... [*looks at the computer screen*] Okay. Then it's 52 divided by ... 100? ... Because ... of the same factor ... ? Er, I think.
- 29. Bill: I understand what you're saying, [*Ann writes 100÷52 in long division form*] there you go. But you keep saying 52 divided by 100, but it's 100 divided by 52 which is the way you're writing it, which is correct.
- 30. Ann: Yeah. I know [grins].
- 31. Bill: Okay. Makes a little bit of difference I think.
- 32. Ann: [*Calculates 100÷52.*] 1.9.
- 33. Bill: Okay.
- 34. Ann: [*Writes "1.9 sec" and circles the 1.9*] And that's again the seconds that it would take him just to go that way [*waves thumb in Turtle's "over"* direction].
- 35. Bill: Good. Very good.
- 36. Ann: So ...
- 37. Bill: Okay.
- 38. Ann: ... we take away ... [*writes "5 sec 1.9 sec =" in column form*] 1.9 ... and, we will come up with how many seconds we will need to fill.

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- 39. Bill: Uh huh.
- 40. Ann: So [pause]. And ...
- 41. Bill: You could use the calculator if you'd like.
- 42. Ann: [*Ignores Bill's suggestion; calculates 5-1.9 on paper; writes "3.1" and circles it*] So, we need to fill up 3.1 seconds [*looks at Bill*].
- 43. Bill: Okay [nods].
- 44. Ann: And to fill up 3.1 seconds and we need to divide that by 100 [looks at Bill].
- 45. Bill: Do you want to show me [points to Ann's scratch paper] how you do that?
- 46. Ann: Or ... I said it backwards.
- 47. Bill: [Chuckles] Yep.
- 48. Ann: Divide a hundred into 3.1 [looks at Bill]. No.
- 49. Bill: No [chuckles].
- 50. Ann: 3.1 into 100 [writes 100÷3.1 in long division form].
- 51. Bill: There you go.
- 52. Ann: [*Calculates 100÷3.1.*] 3.1. And that's 32.2. And ... this is ... distance. So this is speed [*writes "speed" after 32.2, "sec" below 3.1, and "dis" next to 100. Circles "32.2 speed"*].
- 53. Bill: Very good. I like the way you label those. That's really neat.
- 54. Ann: That's the answer. I think.
- 55. Bill: We'll find out here in a minute.
- 56. Ann: So ... 32.2 [*types "32.2" into the Turtle-Back Box. Activates the race*]. The turtle's gonna be fast the first time. [*The turtle and rabbit tie at 5 sec.*]

00:27:12 to 00:27:40

Bill: That looked pretty close. But you're right. You see the overall thing [points to the Time Counter, which reads 5 sec.] that we're looking for, the thing that determines the winner of the race, is the 5 seconds. Well good. You want to put that one in there [points to Activity 3]? You solved another one [Ann writes "32.2" in the <-Turtle box, line 3, Activity 3]. Now we're going to

make just one change on that. Okay, where it says options [*pulls down the Options menu*]. We're going to come down and set the distance.

00:27:40 to 00:32:21

Distance one way is 200 ft. now. Ann does all appropriate calculations.

- 1. Ann: Why do we want to set it?
- 2. Bill: We want to change it.
- 3. Ann: No we don't.
- 4. Bill: [*Chuckles*.] Why? You like dealing with 100? No, that's not for one of there. We're just gonna change ...
- 5. Ann: You want 200 now. That's not fair.
- 6. Bill: Yeah, it's going to go over and back in 200 [changes the one-way distance on the Over and Back program to 200 ft].
- 7. Ann: Wait, but that means one way is 200 and back is 200?
- 8. Bill: That's right [nods].
- 9. Ann: So it's 400!
- 10. Bill: That's correct [nods]. So what I want to do is skip down here to this one [points to the fourth line of Activity 3], you see over here in the right hand side, where it says the 200? [Ann nods]. And, you'll notice, well, the one you just finished was here [points to problem one, Activity 3, where the Turtle speeds are the same as problem four. The only difference in these two problems is the distance for one is 100 and four is 200].
- 11. Ann: The same thing as this [points to problem one].
- 12. Bill: That's correct.
- 13. Ann: It's just this [points to previous problem] with 200.
- 14. Bill: Uh huh.
- 15. Ann: So ...
- 16. Bill: Let's ...
- 17. Ann: ... you have ...

- 18. Bill: Go ahead.
- 19. Ann: 200 ... 40, 20 and 200. [*Pause*] And you would divide [*pause*], um, 40 into a hundred and 20 into a hundred.
- 20. Bill: Into 200 did you say?
- 21. Ann: Yeah [nods].
- 22. Bill: For the first one? Okay.
- 23. Ann: Or you could just add them up and divide.
- 24. Bill: Can you?
- 25. Ann: No. But you could. You can't in this problem [points to Activity 3].
- 26. Bill: If you did would it be correct?
- 27. Ann: No.
- 28. Bill: And do you know why?
- 29. Ann: Because 60 is not the total distance ... er speed, or whatever.
- 30. Bill: Yeah. You can't use an average speed is what you're saying.
- 31. Ann: Yeah.
- 32. Bill: Okay. Do you understand why that's so?
- 33. Ann: No [shrugs shoulders].
- 34. Bill: That's a pretty complex idea, to be honest with you. Okay, so you would do what now. We don't need to go all the way through this. In fact I would like you just to tell me how you would do it.
- 35. Ann: Okay, what you would do is
- 36. Bill: Let's, first of all review the problem. We have the turtle going over at 20
- 37. Ann: And coming back at 40.
- 38. Bill: Coming back at 40. The distance is now going to be a total of 400 feet.
- 39. Ann: [*Ann types "20" into Turtle-Over Box and "40" into Turtle-Back Box*] We have to find out ... the rabbit's speed.
- 40. Bill: And we've got to figure out the rabbit's speed to make that work. Okay.
- 41. Ann: Okay.

- 42. Bill: If we do that
- 43. Ann: So, I'll show you.
- 44. Bill: Go up here for a second [*picks up Activity 3 sheet*]. See what we did up here [*points to problem one*]?
- 45. Ann: Uh huh.
- 46. Bill: Where we had the rab-- the turtle going over at 20, coming back in 40, and the rabbit was set for 26.6. Okay?
- 47. Ann: Ohhh! [Looks up at Bill] Just times 26.6 by 2.
- 48. Bill: ... I don't know, maybe we should go through and work the problem and see if it comes out to that [*Ann chuckles*]. It would be kind-of interesting, really. Well, what I was going to ask you to do is to just walk me through, now, how you were going to do this one [*points to problem four*].
- 49. Ann: Okay.
- 50. Bill: Okay?
- 51. Ann: Okay. First I would divide [*points to the Distance box*] 200 by [*points to the Turtle-> box*] 20.
- 52. Bill: Okay, the turtle's speed over [*points to the computer's Turtle-Over Box*]. And you're going to get 5 when you do that.
- 53. Ann: Yeah, and I would get 5.
- 54. Bill: Uh huh.
- 55. Ann: And then I would divide 40 by 200 or 200 by 40; the same thing.
- 56. Bill: Okay. And do you know what you're going to get?
- 57. Ann: Okay, and I would get 5.
- 58. Bill: Forty into, no, 200 into, 200 ... I'm sorry. I'm misleading you. The [points to the Turtle-> box] 20 into the [points to the Distance box] 200 would be 10. I'm still thinking about ...
- 59. Ann: And that would be 5.
- 60. Bill: Okay.
- 61. Ann: Then I would add those together.
- 62. Bill: Forty into ...

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63.	Ann:	No, no.
05.	<i>1</i> 11111.	110, 110.

- 64. Bill: the 200 ...
- 65. Ann: Yeah.
- 66. Bill: ... is going to give you the 5.
- 67. Ann: 5.
- 68. Bill: Okay.
- 69. Ann: And then I would add 10 and 5, 10 and 5 together [*writes 10+5= in column form*].
- 70. Bill: Okay.
- 71. Ann: To get the average of each, or something like that. I don't know. Um, 15 [*writes "15"*].
- 72. Bill: Is that the average or the total?
- 73. Ann: No, that's the total.
- 74. Bill: Okay. Total time for ... what?
- 75. Ann: For just the turtle alone.
- 76. Bill: To go over ...
- 77. Ann: It would take him 15 seconds.
- 78. Bill: Over and back?
- 79. Ann: ... and back.
- 80. Bill: Okay [nods]. Now what do we do?
- 81. Ann: Then I would do the same thing all over again ... by taking these seconds [points to the 15] and ... dividing them into ... 400.
- 82. Bill: Uh huh.
- 83. Ann: 400.
- 84. Bill: Good.
- 85. Ann: And then I would come up with ... a speed and that would be it at that speed.
- 86. Bill: Very good. Why don't you divide 15 into 400 and let's see what that speed comes up to be.

87. Ann: Okay. [*Calculates 400÷15*] So it's 26.6.

00:32:21 to 00:33:40

Bill attempts to draw out the principle that if you double the distance each animal travels then each animal will travel for exactly double the time, so they will still tie at the same speeds. Instead, he spoke about how doubling the distance will simply double the time it takes them to travel that distance if they keep the same speed, but he did not talk about this meaning that the animals will still tie at the same speeds as before.

- 1. Bill: Hm. Does that mean something to you? Look at the speed up there [points to problem one which has identical speeds but a different total distance] ...
- 2. Ann: [Looks down at Activity 3] So it's the same thing.
- 3. Bill: Yeah, interesting. Same speed ...
- 4. Ann: It should be that you just divide by 2 and it should just be 26.26. But everybody over the years, since the monks screwed it all up. I know they did, they wrote it down wrong.
- 5. Bill: Think about it for a minute. If we have the rabbit and the turtle running, let's take one of them for the minute [*puts left hand down on the table representing a distance line*]. And if we have him going over [*waves right hand over*] and the distance is only 100 feet [*touches the right end of the left hand distance line*] and we've got him running at [*gestures to Ann's papers*] 50 feet per second, or whatever it is, it's going to take him how long to go there?
- 6. Ann: Ohhh ...
- 7. Bill: If it's going to take him 50 feet per second and it's 100 feet [gestures across his distance line]?
- 8. Ann: Uhh, it would take him 2.
- 9. Bill: Two seconds. What if I doubled the length [gestures over the distance line to a point twice his hand length away]?
- 10. Ann: ... It would take him [pause]
- 11. Bill: Just ----
- 12. Ann: ... 4 seconds.

- 13. Bill: Yeah, it's 4. But I haven't changed his rate of travel have I?
- 14. Ann: No.
- 15. Bill: In the mean time if the other one is running, the turtle, let's say, is running along side, all we've done is change the distance [*moves hand out to represent a further distance*] that they have to run, but the distance has been changed the same [*moves both hands as if they are racing*].
- 16. Ann: Oh, oh.
- 17. Bill: Okay.
- 18. Ann: I get it [nods].
- 19. Bill: So, by increasing this length [*drags finger across computer's distance line*] out to 200 [*gestures to a point another screen length away from the 0 ft mark*] now, we've simply doubled the time it takes him at some speed [*gestures to the computer's speed boxes*] to get to the end [*gestures along the distance line*].
- 20. Ann: Oh, okay.
- 21. Bill: Got it? Well good.

00:33:40 to 00:38:53

- 1. Ann: [*Types "26.6" into the Rabbit-speed Box*] So that ...
- 2. Bill: Do you want to write that in there on that worksheet [*gestures to problem four*] then because we did do that one problem. Oh, you're going to try it, good for you.
- 3. Ann: [Activate the race, then she writes "26.6" as her answer for the current problem; race ends in a tie.]
- 4. Bill: Boy, you've got that down cold. Good for you. Good [*picks up Activity 3*]. We're going to get short on time here, so I want to make sure we've gone all the way through this. Uh, tomorrow, we're not going to meet tomorrow by the way because we have a SIP day, a half day.
- 5. Ann: Yeah.
- 6. Bill: So we don't have time to come in and do this.
- 7. Ann: Where do I go? What do I do?

- 8. Bill: Just go to your class.
- 9. Ann: [Whining:] Ohhh ...
- 10. Bill: Oh, this is more fun, huh [*laughs*]?
- 11. Ann: I want to come here. I'll come here anyway.
- 12. Bill: Okay. Umm, we've essentially already done these [gestures to the word problems below the turtle and rabbit rows and columns], but let's go through and just see if you can kind of tell me what it says, or what you would say. Umm, we've figured out these [gestures to the turtle and rabbit rows and columns], for the most part you know how to do all these, so I'm not worried about having you do the last two [note: problems five and six] there right now. It says [reading from first word problem], "Describe the arithmetic you will do tomorrow when you are given the turtle's over-speed, the turtle's back-speed, and the length of the track, and you are asked to enter a number for the rabbit's speed that will make the turtle and rabbit tie." Now that's exactly what you did here [gestures to problem lines one to four]. Okay. But the point of this is, hopefully tomorrow, for example, can to ask you without even saying ...
- 13. Ann: How to do it [gestures to Activity 3].
- 14. Bill: How to do it. And you'll be able to tell me, I think [*shrugs shoulders*], I'm pretty sure you can.
- 15. Ann: Okay [nods].
- 16. Bill: Okay, and then this [*gestures to word problem two*] is the same kind of question. So if nothing else, tomorrow if I see you in the morning we'll sit down and talk about this for a couple of minutes anyway. Because I'd hate to see all of this go to waste, if you forget it, but I don't think you're going to forget it.
- 17. Ann: But why aren't we coming tomorrow?
- 18. Bill: Tomorrow, because it's only 20 minutes long. By the time we get here and get ready ...
- 19. Ann: Yeah, but we don't have to record or anything we could just do stuff [*chuckles*]. I don't know.
- 20. Bill: But I don't have anybody to cover my class, see?
- 21. Ann: You don't?
- 22. Bill: Because one of the people from this group is covering my class so I can come over here, and he won't be here tomorrow [*chuckles*]. I'd like to do it with

you. We can come over here some other time if you want. You know, we can't do it during that class time. Okay, here's what I'd like you to do for homework tonight. Two problems on here [*hands Ann Activity 4*]. The top one says ...

- 23. Ann: Homework?! But I can't do homework [*Bill chuckles*] if I'm not going to see you!
- 24. Bill: Well, I'll see you tomorrow.
- 25. Ann: Oh, no you won't.
- 26. Bill: Believe me. Okay, Activity 4 here: "Bill traveled 35 mph for a 100 miles and 44 mph for 50 miles."
- 27. Ann: So it's just the same thing except you're talking about people and cars.
- 28. Bill: Yeah, but it's a little bit different, too. There's a twist in there. It says he traveled 35 mph for ... how far?
- 29. Ann: One hundred miles.
- 30. Bill: A hundred miles.
- 31. Ann: And 44 miles.
- 32. Bill: And he then traveled 44 mph for 50 miles. "What questions can you answer about Bill's trip?" So I want you to ask yourself the questions and then answer them. Okay?
- 33. Ann: How many of these questions do I have to make up [chuckles]?
- 34. Bill: Well, what questions can you answer, er, ask about that?
- 35. Ann: What's the total miles it takes, he took? How long did it take him to get the total miles?
- 36. Bill: Okay, anything else?
- 37. Ann: Why did he have to go at those speeds?
- 38. Bill: [*Laughs*] Well that we don't have to worry about because we don't have anybody to answer that one. But couldn't we also ask [*points to the problem*], for example, umm, how long did it take him to go 100 miles at 35 mph?
- 39. Ann: Yeah.
- 40. Bill: Okay. And how long it takes him to go the second one. Because you were going all the way to the end and saying, "Well how long is it going to take him to get the total trip?" Okay?

- 41. Ann: Okay.
- 42. Bill: So those are the kinds of questions, that's fine, just jot down briefly your questions. They don't [*shakes head*] have to be, you know, in paragraph form. Write down your questions and see if you can calculate them.
- 43. Ann: Okay.
- 44. Bill: Okay? Second part down here [*gestures to the second problem*]: "Sue paid \$9.46 for Yummy candy bars at \$0.43 per bar, and she paid \$6.08 for Zingy candy bars. Sue bought 38 of these candy bars. What was the price of a Zingy candy bar?" That should be plural here: "Zingy candy bars." Now you don't have to ask yourself the questions on that, you just have to answer the question. Do you think you could do that?
- 45. Ann: Yeah.
- 46. Bill: No problem. Do you have any questions?
- 47. Ann: What's behind that?
- 48. Bill: [*Moves Activity 4 out of the way, to reveal scratch paper*] Oh, that's just a place to do work.
- 49. Ann: Oh, okay.
- 50. Bill: Okay. But on this you could just do these right here [gestures to the blank spaces below the two problems].
- 51. Ann: Okay.
- 52. Bill: [Hands Activity 4 to Ann] Do you have any questions ...?
- 53. Ann: Am I coming here next week, or not?
- 54. Bill: No [shakes head].
- 55. Ann: No. Okay.
- 56. Bill: This is a trial run, so to speak. It's giving both of us a chance to try this program out because I've never worked with the program before either. And we're hopefully going to be able to use this next year. And you will be one of the experts that's already knowing how to do everything. Any other questions on it?

00:38:53 to 00:39:15

- 1. Ann: Yeah. How did he make it up?
- 2. Bill: How did he write the program?
- 3. Ann: Yeah.
- 4. Bill: Well you mean as far as actually writing the program code in the computer, I don't know how he did that because, I haven't actually seen the program. But do you mean the actual information that's in here or how to come up with the idea of doing something like this?
- 5. Ann: How did he come up with the idea of just doing it?

00:39:15 to 00:41:32

- 1. Bill: Okay. The concept of speed which is distance divided by time that you learned in science,
- 2. Ann: Uh huh.
- 3. Bill: ... is a difficult concept for a lot of people to understand. Sixth, seventh graders especially because we're talking about a ratio of two numbers that are independent of each other. And by that I mean on one thing you have speed, and on the bottom side of this fraction, some people call it a fraction although it is a ratio [uses hands to show a fraction], you have, um distance ... I'm sorry, distance and time. And those are two entirely different units. And trying to work with them, as you found out the first or second day we were working here, "Uh-oh something is kind of different here, it doesn't seem to work right." So the reason for the program is to help teach what that relationship is. That we're dealing with distance and time [makes an *imaginary distance line with his hand*]. If we're going to divide a distance of 200 feet up [uses his other hand to make pretend tick intervals in his left hand into 5.2 seconds of time, that going up proportionally and dividing up the distance into 5.2 segments will give me the speed. That's the part that's hard to understand.
- 4. Ann: Okay.
- 5. Bill: But you're a past expert on that now, see? [*Chuckles*] Any other ones?
- 6. Ann: Yeah. Do we have to do algebra?
- 7. Bill: Oh, you'll like algebra.
- 8. Ann: You mean we get to do it?

- 9. Bill: Yeah.
- 10. Ann: Good.
- 11. Bill: You're going to be doing algebra in the eighth grade.
- 12. Ann: I like algebra.
- 13. Bill: You may not even realize it before, but you're actually dealing with algebra here [gestures to Ann's papers].
- 14. Ann: Really?
- 15. Bill: Uh huh. The only reason people think algebra is so strange is because we start using letters to represent numbers. But if I was to write this in an algebraic form for you, it wouldn't be any different. Ann, you did--done good, as we say. You did very well. You should get your mom to pat you on the back and give you an ice cream cone.
- 16. Ann: Okay [closes the Over and Back file].
- 17. Bill: In fact, if I could buy you an ice cream today, I'd buy one. But I don't have any ice cream around here. And I thank you very much.