Calculus Learning and Teaching Around the World

An event hosted by Pat Thompson and Guershon Harel on 9-10 June 2021 to celebrate a ZDM Special Issue!

IRENE BIZA MAY 18, 2021 05:34PM

Welcome!

Add your question anywhere on this blackboard!
First, choose **which paper** of the SI your question concerns. Then, **type your question** in the comments section for that paper!
Alternatively, **use the "+" button** at the bottom right of the screen to add a comment or question that is about more than a paper/the entire SI or raises a broader issue.
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Pat Thompson & Guershon Harel: Ideas Foundational to Calculus Learning and Their Links to Students' Difficulties

You emphasize "variation in bits" as a foundational idea students should develop in school. Why is "in bits" so important? — ANONYMOUS

You are discussing Cornu's ideas about communication in everyday life. Natural language about the concept of limit requires implicitly the variable time; that is, approaching as getting closer and closer (progressively) to un value. This natural language is related to potential infinity. When formalizing the concept of limit, we are using actual infinity, the elements of a sequence, for example, do not move. How to cop this problem in the mathematics classroom? — ANONYMOUS

You discuss differentials. There are two different prominent conceptions of differentials - the one originating with Cauchy, in which dx varies, and the one originating with Leibniz and given rigorous foundations by Robinson, in which dx is a fixed infinitesimal. Can you address these two visions, the extent to which they are compatible or not, and how you envision the role of each (or both) in Calculus? — ANONYMOUS

It is true that Calculus is used and taught differently in different educational systems. Sometimes we say Calculus and we mean different things even in the same educational context. Your SI helps to map out such differences and to build a common language. After the experience of this special issue how would you classify the meanings of Calculus in the literature and the corresponding systems you have visited? — IRENE BIZA

Given your (earlier) interest in younger students' mathematical experiences, as you state at the start of your video, can you say a few words about how we can engineer early experiences of variable and real number so that students are more smoothly prepared for dealing with variables and real numbers as per your proposition? — ANONYMOUS

David Bressoud: The Strange Role of Calculus in the United States

Thank you for your excellent article. Your stance seems to be that U.S. students should wait until college/university to study calculus. But many other countries include substantial ideas of calculus in their high school curriculum. What is different about the U.S. that students should not study calculus in high school? — ANONYMOUS

Reply from David Bressoud: I have no trouble with teaching calculus in high school provided that access is not determined by socio-economic status AND that universities modify their own curricula and pedagogical supports so that they articulate with what is learned in high school and do not disadvantage students who did not have access to calculus in high school. — ANONYMOUS

There is a debate on what preparation students need at school level towards studying calculus at tertiary level; some teachers say basic algebra will be sufficient and others advocate for emphasis on Calculus (in some cases with inclusion of definitions and proof). In your view, what might be an appropriate preparation for students at school level? — IRENE BIZA

Do the inequities that your article outlines so eloquently for the USA appear in the same or similar manner around the globe? In other words, how is the US case placed in the global landscape? — ANONYMOUS

Tin Lam Toh: School Calculus: National Education Agenda versus Preparing Students for Undergraduate Mathematics Education

Question: It seems from the paper that there is a "gap" in the transition of calculus from secondary level to the pre-university level. Could you explain a little more? — ANONYMOUS

What is your stance on students doing calculus before college/university? — ANONYMOUS

You propose a sound pedagogy for teaching calculus that will facilitate learning beyond the procedures. What the characteristics of this pedagogy might be and how it would address the needs of students with different aspirations (e.g., students who aspire to study engineering or economics or mathematics, etc.) — IRENE BIZA

Further to Irene Biza's question, how may your sound pedagogy play out in the experiences of students who may not wish to pursue STEM studies or even not go to university at all? — ANONYMOUS

How may you explain this double phenomenon: (1) Singapore students are known to be among the best from the world (PISA and TIMSS) and (2) your surprise about their entry level (to the university) of school Calculus knowledge? — ANONYMOUS

Kristin Frank & Pat Thompson: School Students' Preparation for Calculus in the United States

the place of differential equations in the mathematics and physics high school program in united states. — ANONYMOUS

Your analysis revealed a disconnect between meanings productive for learning calculus and the meanings held by U.S. high school teachers and it also revealed that many teachers experience a continuity of meanings between their university experiences and the high school classrooms as teachers. Do you mean that: (1) there is no difference between university calculus and calculus to be taught by the teacher? (2) university mathematics do not give future teachers opportunities to construct productive meanings for calculus notions? — ANONYMOUS

The second discontinuity of Klein (1908/1932) explains that high school teachers face a discontinuity when they move from university calculus to high school calculus. Accordingly, how do you explain the continuity of meanings that US high school teachers experience from university to calculus classrooms? — ANONYMOUS

Your study highlights the importance of teacher professional development on Calculus. How would you envisage a professional development that has the affordances to support the teachers to overcome the challenges with the teaching of Calculus? — IRENE BIZA

(Elena Nardi, UK): Across your article, the notion of "productive meaning" is ubiquitous. Can you explain (in a nutshell) your working definition of this notion? Also, can you say a little more about the choice of South Corea as a component of your comparison with the case of the US? — ANONYMOUS

Imène Ghedamsi & Thomas Lecorre: Transition from High School to University Calculus: A Study of Connection

How did you identify the main learning aspects that could potentially damage the passage between school calculus and university calculus? — ANONYMOUS

Why should high school teachers adjust or change their teaching actions? Why are they supposed to adhere and adapt their instruction according to what is valued at the university? Is it that university teachers do not worry about improving their knowledge of teaching calculus? — ANONYMOUS

Can you tell us a bit more about the 'guided reflection approach' in step 3? — IRENE BIZA

Your observation that the Tunisian case does not fit the narrative of transition from school to university mathematics as a shift from informal to formal mathematics is pertinent. There are other cases such as this. Can you say a little more about how Tunisia fits this global landscape? — ANONYMOUS

Michael Tallman, Zackery Reed, Michael Oehrtman, & Marilyn P. Carlson: What Meanings Are Assessed in Collegiate Calculus in the United States?

Impressive study. Do you know (or would you venture a guess) whether the percentage of tasks in homework assignments that require understanding is higher than in exams? — ANONYMOUS

What advice would you give to mathematics faculty who want to improve the assessment practices of graduate teaching assistants? — ANONYMOUS

Your (seven) impressive conclusions can drive recommendations for what needs to change in curricular materials, classroom activities and (eventually) form/content of exam questions. You reached these robust conclusions (also) because you operationalised what you were looking for ("understanding") concretely and clearly. Thank you. Could this future operationalisations include something also about mathematical meanings in extra-mathematical situations? — ANONYMOUS

Rob Ely: Teaching Calculus with Infinitesimals and Differentials

Interesting and compelling argument for differentials-based calculus. How likely is it that students with poorly formed concepts of the numerical continuum, especially rational numbers, can conceive of differentials as composing an interval? — ANONYMOUS

Are you suggesting eliminating the theory of limits altogether? If not, where in the curriculum should this be introduced. — ANONYMOUS

Does using differentials to introduce calculus change the approach to problem solving in any fashion? — ANONYMOUS

Would teaching differentials necessarily be a departmental decision? Would students be harmed if some faculty are teaching differentials while others are not? — ANONYMOUS

How do such approaches to Calculus - with infinitesimal increment of a variable quantity - address the needs of non-mathematics specialist? — IRENE BIZA

(Elena Nardi, UK): Your proposition for working with differentials, rather than limits, seems to be closer to the experiences of quantitative reasoning that students may have in school. Do you agree with this observation? — ANONYMOUS

As mentioned in a previous comment, you present a compelling argument for a differentials-based calculus. The potential gains are clear from your presentation. But what might be the drawbacks of such an approach? — ANONYMOUS

Ricardo Scucuglia R. da Silva, Lara Martins Barbosa, Marcelo C. Borba, & Andre Luis Andrejew Ferreira: The Use of Digital Technology to Estimate a Value of Pi: Teachers' Solutions on Squaring the Circle in a Graduate Course in Brazil

Can you share any insights of how the teachers deployed the three solutions in lessons and how students responded to those? — ANONYMOUS

Luis Moreno-Armella: The Theory of Calculus for Calculus Teachers

You 'confront' students with functions like $f(x) = x \sin x$, to evoke cognitive conflict in the students. Can you say more about what conflict this is, and how it makes the students ready to learn the formal definitions? — ANONYMOUS

Fernando Hitt & Sarah Dufour: Introduction To Calculus Through An Open-Ended Task In The Context Of Speed: Representations And Actions By Students In Action

What foundational ways of thinking with graphs and quantities do you think the students were lacking, that would account for the difficulties they had when interpreting kinematic phenomena graphically? — ANONYMOUS

How familiar was the kinematic context to the students from other types of tasks in the past? — ANONYMOUS

Gilbert Greefrath, Reinhard Oldenburg, Hans-Stefan Siller, Volker Ulm, & Hans-Georg Weigand: Basic Mental Models of Integrals: Theoretical Conception, Development of a Test Instrument, and First Results

Thank you for this careful study. One small question: Just before the Discussion part, you say that the scores vary significantly between students with high and low math performance (and that the effect is strong for the AR BMM and weak for the AV BMM). Which direction are these effects? What are they for the other two BMMs? Can you conjecture about why these effects might be this way? — ANONYMOUS

Thanks for the question, indeed there are some interesting observations to be made. AR: average BMM score is 2.71 for weaker and 2.96 for stronger students (significant)AC: 2.16 vs. 2.01, almost significant (p=0.052). For RE and AV there is no significant difference, but for all 4 BMMs the direction is the same: Better students have better BMM scores. This is of course what should be expected but it is a bit disappointing that the effect I not stronger. This may be the case because many tasks in school focus on simple calculations that can be done without evoking a particular mental model. However, the order of the effect is as expected: As AR is still dominant in schools it should not come as a surprise that having this model developed gives an edge in examinations. (Reinhard Oldenburg) — ANONYMOUS

Hyunkyoung Yoon, Younggon Bae, Woong Lim, & Oh Nam Kwon: A Story of the National Calculus Curriculum: How Culture, Research, and Policy Compete and Compromise in Shaping the Calculus Curriculum in South Korea

Does your country have a committee like national academy so that the group of reputable scholars in math education can influence the curricular policy? It is amazing calculus becomes the gatekeeper when it is an open gate to great sciences. — ANONYMOUS

Appreciated your presentation. An interesting example of an intersection between mathematics education and politics. — ANONYMOUS

How is the teaching of Calculus in South Korea, especially in terms of students' learning and exam anxiety, six years after the implementation of the reform? Did the reform make any difference? What would you suggest as a useful theoretical approach for the evaluation of the impact of this reform? — IRENE BIZA

Was / Is there any debate in Korea about how calculus notions that may so far have been experienced as challenging (e.g. the definition of integrals as limits) can return in the curriculum but introduced in innovative ways that engage the students and do not shy away from the complexity of the subject? — ANONYMOUS

Tommy Dreyfus, Anatoli Kouropatov, & Gila Ron: Designing a High-School Calculus Curriculum – The Case of Israel

Very interesting story of how the Israel 5-unit curriculum was developed. Can you now see a way in which maths education research can have a more systematic impact on curricula documents? Your account seems to be that it was by chance that people on the committee were aware of maths education research. — ANONYMOUS

What is your first experience after the first implementation of the new curriculum in Israel? — IRENE BIZA

What insights can you share in light of implementation of the new Calculus curriculum so far and from the synergy between education professionals and mathematics education researchers? — ANONYMOUS

Guershon Harel: The Learning and Teaching of Multivariable Calculus: A DNR Perspective

I have trouble imagining a student being prepared for the multivariable curriculum you outline in a standard U.S. calculus curriculum. What implications do you see for earlier calculus curricula for students to be prepared for a course emphasizing the ideas you do? — ANONYMOUS

I am fascinated by the construct of a curtailment that maybe an inhibition or a catalyst and I can see its potential to be applicable in other mathematical domains such as Group Theory where curtailments in notation and in meaning/actions are also frequently present. Did this construct come out of analyses of calculus related data or out of analyses from other mathematical domains? — ANONYMOUS

Rafael Martinez-Planell & Maria Trigueros: Multivariable Calculus Results in Different Countries

You talk about having obtained very good results in a third cycle of research, in terms of having students show they were able to construct a process conception for two-variable functions. Have these results been replicated elsewhere? — ANONYMOUS

You mention using the "locally linear" approach to the teaching of the differential multivariable calculus. Yet, in your results you show that students don't seem to make the expected constructions of directional derivative and total differential. Can you talk a little bit more about this? — ANONYMOUS

It seems that in your research you are not making much use of technology for teaching multivariable calculus. Could you comment about this? — ANONYMOUS

What role does the genetic decomposition play in your research? How does it compare with a learning trajectory? — ANONYMOUS

Would you suggest teaching multivariable calculus at high school level? — IRENE BIZA