Round-off Error and the Floor Function

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Why does this happen?

$$\text{GC's computation of } \left\lfloor \frac{2.72}{0.01} \right\rfloor \text{ is 272 (which is correct).}$$

$$\text{GC's computation of } \left\lfloor \frac{4.72-2}{0.01} \right\rfloor \text{ is 271 (which is incorrect).}$$

The explanation has to do with the fact that computers have a finite number of digits with which to represent all real numbers, and it represents all numbers in base 2. So any number that is not a sum of powers of 2 is actually approximated in the computer's memory.

0.01 (base ten) is represented in the computer as:

0.00000010100011110101110000101000111101011100001010001111011 (base two)

which, in base ten, is actually

0.01000000000000000020816681711721685132943093776702880859375 (base ten).

So, 0.01 in the computer is a little more than 0.01.

2.72 (base ten) in the computer is represented in the computer as:

10.101110000101000111101011100001010001111010111000011 (base two)

which, in base ten, is actually

2.720000000000000195399252334027551114559173583984375 (base ten).

So, 2.72 in the computer is a little *more* than 2.72.

4.72 (base ten) in the computer is

100.10111000010100011110101110000101000111101011100001 (base two)

which, in base ten, is actually

4.71999999999999975131004248396493494510650634765625 (base ten).

So, 4.72 in the computer is a little *less* than 4.72.

Therefore, 4.72 - 2 is actually represented in the computer as:

4.719999…25 – 2, or 2.719999…25 (base ten), while 2.72 in the computer is represented as 2.720000000…75 (base ten).

$$\text{The computer's computation of } \left\lfloor \frac{4.72-2}{0.01} \right\rfloor \text{will be } \left\lfloor \frac{2.719999...25}{0.0100...11} \right\rfloor = 271\,.$$

$$\text{The computer's computation of } \left\lfloor \frac{2.72}{0.01} \right\rfloor \text{will be} \left\lfloor \frac{2.72000...75}{0.0100...11} \right\rfloor = 272\,.$$

The computer is not broken. It just doesn't have enough digits to represent all real numbers exactly.